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Social contagion and the survival of diverse investment styles ☆

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ABSTRACT

We examine the contagion of investment ideas in a multiperiod setting in which investors are more likely to transmit their ideas to other investors after experiencing higher payoffs in one of two investment styles with different return distributions. We show that heterogeneous investment styles are able to coexist in the long run, implying a greater diversity than predicted by traditional theory. We characterize the survival and popularity of styles in relation to the distribution of security returns. In addition, we demonstrate that psychological effects such as conformist preference can lead to oscillations and bubbles in the choice of style. These results remain robust under a wide class of replication rules and endogenous returns. They offer empirically testable predictions, and provide new insights into the persistence of the wide range of investment strategies used by individual investors, hedge funds, and other professional portfolio managers.

1. Introduction

The Efficient Markets Hypothesis (Samuelson, 1965; Fama, 1970) maintains that market prices fully reflect all publicly available information. It is based upon the premise that there are market participants who will take advantage of any mispricing, and that investors with correct beliefs will grow richer at the expense of agents with incorrect beliefs (Fama, 1965). In consequence, markets will be dominated by agents with accurate beliefs about prices (Alchian, 1950; Friedman, 1953).

However, an accumulation of evidence from psychology, cognitive science, behavioral economics, and finance has documented significant violations of individual rationality and the Efficient Markets Hypothesis. In particular, there is evidence of social contagion of investment behavior in financial markets that is not always explained by rational information processing.[4] To better understand

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[4] Examples of social contagion include evidence from stock markets (Hong et al., 2004; Ivković and Weisbenner, 2007; Brown et al., 2008; Kaustia and Knüpfer, 2012; Ozsoylev et al., 2014; Ammann and Schaub, 2021), mutual funds and hedge funds (Hong et al., 2005; Cohen et al., 2008; Boyson et al., 2010; Pool et al., 2015; Kuchler et al., 2022), and housing markets (Burnside et al., 2016; Bailey et al., 2018); see also the review of Hirshleifer and Teoh (2009) and the discussions of social economics and finance of Shiller (2017) and Hirshleifer (2020).

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the dynamics of market contagion, the Efficient Markets Hypothesis can be complemented by the Darwinian perspective of natural selection. The application of natural selection to economic thought extends back to the 1950s (Alchian, 1950; Penrose, 1952; Friedman, 1953). More recently, the Adaptive Markets Hypothesis (Lo, 2004, 2017) uses an evolutionary perspective to reconcile economic theories based on the Efficient Markets Hypothesis with behavioral economics.[5]

In this article, we model the transmission of ideas between investors in order to analyze the evolutionary survival of competing investment styles. Motivated by the model of Brennan and Lo (2011), we consider a market in which each investor has a propensity to invest in one of two investment styles.[6] We refer to this propensity as the investor's *investment philosophy*. Investors with higher realized returns produce more "offspring" in the next period of the model by transmitting their ideas to other investors via social interaction. Selection results in the differential survival of investors' behavioral traits, i.e., their investment philosophies.

The distinction between *investment style*, a specific trading behavior, and *investment philosophy*, a general approach to investing, is much like the military distinction between specific tactics and general strategy. An example of an investment style is holding value stocks (i.e., stocks with high book-to-market ratios), or holding momentum stocks (i.e., stocks that have experienced high returns in the past 12 months). An example of an investment philosophy is the general approach of buying cheap stocks, or buying stocks with prospects for growth, where the investor uses discretion in defining "cheap" or "prospects for growth." An investor with the philosophy of buying cheap stocks might on occasion feel that a rapidly growing firm such as Amazon is still cheap in price relative to its prospects, and therefore might sometimes invest in what is usually regarded as a growth stock.[7]

We demonstrate that heterogeneous investment styles are able to coexist in the long run, implying the survival of a more diverse set of strategies than occurs in traditional portfolio theories. For example, under the Capital Asset Pricing Model (Sharpe, 1964), all investors hold the market, and therefore they all pursue the same investment strategy. Under the Intertemporal Capital Asset Pricing Model (Merton, 1973), all investors hold the market and a set of hedge portfolios, usually presumed to be small in number, implying only a limited amount of diversity.[8] In contrast, our results are consistent with the stylized fact that numerous competing investment styles coexist in the market. Examples of persistent surviving investment styles include value versus growth, momentum versus contrarianism, diversification versus stock-picking, technical versus fundamental, and so on (Cronqvist et al., 2015; Cookson and Niessner, 2020). We show that the survival of diversity is a consequence of general principles of evolution in the face of risk.

Our model provides a framework for understanding the general multiperiod dynamics of contagion between a pair of competing investment styles. The bulk of the literature on the evolutionary survival of financial trading strategies has focused on the accumulation of wealth by individuals and its impact on trading (e.g., the influence of investors with different beliefs or preferences on prices). We instead focus on evolution via the *contagion* of investment ideas. This focus implies that it is not necessarily the philosophies that promote investor wealth that survive, but rather, the philosophies that are good at spreading. While it is true that the financial performance of a philosophy is one important element in determining its ability to spread, it is not the only one. In contrast to some studies which take this approach (Hirshleifer, 2020; Han et al., 2022), we allow for broadly general probability distributions for the number of offspring in each generation, instead of assuming, for example, a process in which the number of investors of each type changes by exactly one in each generation (Moran (1958)). This generality allows us to characterize the survival of investment philosophies in the long run in relation to the return characteristics of the underlying securities, including their mean returns, betas, and idiosyncratic volatilities.

The model has several testable implications. In the CAPM, the quality of an investment style is often measured by its excess return (alpha) above the market's return at a given level of risk. This suggests that high-alpha strategies tend to survive (at least to the extent that alpha persists over time). However, we find that the survival of an investment style is determined by several elements, including its expected return, beta, and volatility. When determining a strategy's survival with respect to these return characteristics, a style's beta-scaled expected gross return—defined as the expected gross return of a style divided by its beta—plays a critical role. We call this return its *scaled alpha*.

Scaled alpha plays two roles in our model. The first is in determining a non-monotonic relationship between an investment style's beta and its popularity and future survival. In particular, an investment style with low beta is promoted in market evolution only when its scaled alpha is comparable to that of the alternative style. In contrast, when a style has a much higher scaled alpha than its alternative, high beta can promote its popularity. This result implies that a style's scaled alpha, not the traditional CAPM alpha, is a key determinant of the popularity of low-beta investment styles in a population.

The second role of scaled alpha is in determining a non-monotonic relationship between market volatility and the popularity and survival of an investment style. In particular, high market volatility promotes investment styles with high scaled alphas, and is opposed to investment styles with low scaled alphas. A high scaled alpha can therefore be understood as a defensive characteristic of an investment style, in the sense that investors will tend to allocate to styles with high scaled alpha in volatile markets. This can be

[5] See Holtfort (2019), Levin and Lo (2021) and references therein for recent examples of research on the interplay between evolutionary theory and financial market dynamics.

[6] The term "investment style" is used in a growing literature on style investing; see, for example, Barberis and Shleifer (2003), Teo and Woo (2004), Froot and Teo (2008), Kumar (2009), Wahal and Yavuz (2013), and Cronqvist et al. (2015). In our model, for example, an investment style can be represented by a portfolio of stocks with a common attribute such as high momentum.

[7] For example, Lettau et al. (2018) documented that the so-called "value" mutual funds are seldom tilted toward high book-to-market stocks, and are often tilted toward low book-to-market stocks.

[8] This result is an example of so-called *K-fund separation*, in which the optimal portfolios that any investor may hold can always be formed by combining a finite set of K investment funds.

empirically tested by examining shifts in investment style such as value versus growth, momentum versus contrarian, or fundamental versus quantitative as a function of market volatility.

More generally, our model helps to explain and predict the survival of a diverse range of investment styles given their return characteristics. For example, there are numerous categories of hedge funds with widely varying investment styles (Chan et al., 2006). The hedge fund sector is subject to intense selection pressure, and has been called the "Galápagos Islands" of finance (Lo, 2008).[9] Darwin's original 1835 observations in the Galápagos Islands suggested that environmental segmentation was the source of evolutionary diversification. In fact, our framework suggests that diversity can persist even within a single non-partitioned environment, a surprising but important distinction in market evolution.

We check the robustness of the implications of our model by considering several extensions. First, we allow for very general rules of replication that are increasing functions of realized returns, capturing the intuitions that higher returns benefit the spread of an investment philosophy. We find that higher variances of a style promote survival when replication functions are convex such that they represent lottery-like preferences. Second, return distributions are exogenous in our basic model, which we extend by considering market equilibrium with endogenous returns in the spirit of Lux's (1995) classical model. When more investors adopt a philosophy, the demand for the stocks that this investment philosophy calls for buying increases. This demand is cleared in the market with supplies from a group of fundamentalist traders based on price deviations from the fundamental value, thereby setting the actual price. We find that the key implications from our model remain valid under both extensions.

In other extensions of our model, we allow for important psychological forces that affect investor receptiveness toward the investment philosophies of others. The first is conformist transmission (Boyd and Richerson, 1985), the phenomenon that investors view others as being well-informed and therefore that they in turn will follow the choices of these others. We show that conformist preference slows down evolutionary convergence, potentially leading to price deviations from fundamental values and lower degrees of market efficiency, a similar result to Scholl et al. (2021), but through a different channel.

The second psychological force we investigate is attention to novelty, the phenomenon that investors are more likely to pay attention to a novel investment philosophy if it is *very* different from the most popular philosophies. Attention to novelty acts in opposition to conformist preference, and leads to an even higher degree of diversity among investment philosophies in the long run. It generates oscillations and bubbles in prices in certain financial environments, a phenomenon similar to models of herd behavior (Lux, 1995; Chinco, 2023), but again through a different channel. We also propose potential empirical tests for the survival of investment philosophies in relation to different proxies for attention in the empirical finance literature.

2. Literature review

Our model is related to a large literature that uses evolutionary ideas to model the dynamics of financial markets. In classical models, agents are assumed to maximize their expected utility with rational price expectations, but may disagree on the dividend process. Some studies have found that individuals with more accurate beliefs will accumulate more wealth and dominate the economy (Sandroni, 2000, 2005), while others argue that wealth dynamics need not lead to rules that maximize expected utility using rational expectations (Blume and Easley, 1992), and that individuals with wrong beliefs may drive out individuals with correct beliefs, owing to different propensities to save in an economy with growth (Yan, 2008). Heterogeneity may persist when markets are incomplete (Blume and Easley, 2006), when learning does not converge (Sandroni, 2005), when non-accurate beliefs and non-optimal rules interact (Bottazzi et al., 2018), and when agents have recursive preferences (Dindo, 2019). We show that heterogeneity is persistent with return-based contagion dynamics, and is reinforced by psychological effects such as attention to novelty.

A second strand of this literature contrasts itself from the rational expectation paradigm by studying investment heuristics, such as fixed-mix rules and functions of past realized returns.[10] Under this setting, Evstigneev et al. (2002, 2006, 2008) show that the Kelly rule that invests according to the proportions of the expected relative dividends is evolutionarily stable and sometimes also attains the highest growth rate of wealth. The investment philosophy in our model is closer to this second strand in spirit, although we go further to assume that agents use no information at all.

A third strand of this literature concerns the performance of rational versus irrational traders. It has been shown that irrational traders can survive in the long run, resulting in the divergence of prices from fundamental values.[11]

In all of the three strands of literature above, market selection is studied from the perspective of wealth accumulation. However, our framework focuses on evolution via the social contagion of investment ideas, increasingly relevant given recent developments in information technology and the growth of social networks.[12] The reproducing units are not investors or traders, but instead are instances of investment philosophies. As a result, it is not necessarily the philosophies that promote investor wealth or welfare that survive, but rather, the philosophies that are good at spreading. This is analogous to a disease-causing virus that spreads at the

[9] In biological evolution, this island group is a textbook example of the evolutionary adaptation that occurs after a species migrates into multiple segmented environments.

[10] Chapters in the handbook of Hens and Schenk-Hoppé (2009) contain an excellent list of classical models. Other examples include Lensberg (1999), Amir et al. (2005), Hens and Schenk-Hoppé (2005, 2020), Lensberg and Schenk-Hoppé (2007), Bottazzi and Dindo (2014), Palczewski et al. (2016), and Bottazzi et al. (2018). Recent work has focused on the inclusion of short-selling (Amir et al., 2020), endogenizing the dividend process (Evstigneev et al., 2020; Amir et al., 2021), the use of the risk-free asset as a numeraire (Belkov et al., 2020a), game-theoretic properties of survival portfolio rules (Belkov et al., 2020b), and portfolio insurance strategies (Barucci et al., 2021).

[11] See, for example, De Long et al. (1990, 1991), Kyle and Wang (1997), Biais and Shadur (2000), Hirshleifer and Luo (2001), Hirshleifer et al. (2006), Yan (2008), and Kogan et al. (2006, 2017).

[12] See, for example, Shiller (2017), Hirshleifer (2020), Kuchler and Stroebel (2021) and references therein.

expense of its hosts. This different focus motivates us to not only generalize the rules of replication between two generations as functions of realized returns, but also study psychological effects that are not related to returns and wealth at all.

Other models focus on the evolutionary implications for asset prices. Lux (1995) shows that equilibrium prices can deviate from fundamental values with herd behavior. Brock and Hommes (1997, 1998) show that complicated price dynamics such as chaos can emerge in adaptively rational equilibrium. Scholl et al. (2021) show that the convergence to equilibrium (efficiency) can be very slow in market selection. In general, the dependence of investment rules on past prices generates feedback, market instability, and asset mispricing (Hommes, 2006; Anufriev and Bottazzi, 2010; Anufriev and Dindo, 2010).[13] One unique feature of our model is that feedback in the market comes from not only past prices, but also from the behavior of other investors not directly related to prices.

Finally, our model is also related to two recent threads in the behavioral finance literature. The first concerns how investors subject to cognitive limits form beliefs, and its implications for asset prices.[14] The second thread studies how interactions in social networks affect investor behavior and asset prices, including, for example, Han and Yang (2013), Hirshleifer (2020), Han et al. (2022), and Kuchler and Stroebel (2021).[15]

The key difference between our model and this literature is that we explicitly model the replication process due to social contagion and study its evolutionary implications for the survival of philosophies. We allow for general distributions of the number of offspring in each generation, instead of assuming, for example, the normally-distributed dividend processes as in Hong et al. (2007), the Moran (1958) process as in Han et al. (2022), or the DeGroot (1974) model as in Pedersen (2022). This generality allows us to derive a comparative analysis of the statics with respect to the mean returns, betas, and idiosyncratic volatilities of the underlying securities, leading to several useful testable implications.

Our model builds upon the analysis of Brennan and Lo (2011), which develops a binary choice model in order to understand the survival of economic behaviors in stochastic environments. We extend this model to study the contagion of investment ideas, explicitly modeling investment styles in relation to their systematic and idiosyncratic return, and treating investment philosophies as propensities to adopt different styles. We generalize the replication rules to a class of functions of realized returns. We take into account that in equilibrium, changes in popularity of styles will affect their expected returns, and we establish that a mix of investment styles is able to survive in the long run. Finally, we analyze how psychological effects affect the evolutionary survival of competing investment philosophies.

3. A model of competing investment philosophies

Consider two investment styles a and b in discrete time, each generating gross returns $X_a \in (0, \infty)$ and $X_b \in (0, \infty)$ per period. The returns realized in the t-th period are denoted by (X_{at}, X_{bt}). We assume that:

Assumption 1. The returns (X_{at}, X_{bt}) are independently and identically distributed (IID) over time $t = 1, 2, \cdots$, and described by the probability distribution function $\Phi(X_a, X_b)$.

Assumption 2. (X_a, X_b) and $\log(f X_a + (1 - f) X_b)$ have finite moments up to order 2 for all $f \in [0, 1]$.

Next, consider a population of investors, each of whom lives for only one period and makes only one decision: to invest in either style a or b. An investment style can be interpreted as a portfolio of stocks with a common attribute. For example, a could be a high-variance investment style and strategy b could be a low-variance one. Other investment style dichotomies include value versus growth, aggressive versus defensive, momentum versus contrarian, and stock-picking versus diversifying. Each investor's propensity to invest in style a is denoted by $f \in [0, 1]$. This means that the investor chooses style a with probability f, and style b with probability $1 - f$. We will refer to f as the investor's *investment philosophy*.

The investment philosophy is a general approach to investing, whereas an investment style represents the actual trading behavior that the investor follows in a specific context. For example, the value *philosophy* refers in general to buying stocks that the investor regards as a good bargain—relatively cheap compared to their "value," which might be defined in many ways in different contexts. The value *style* is something much more specific, such as trading based on the book-to-market or P/E ratio. The probability f in this example is the probability that the value philosophy investor actually follows the specific strategy of trading based upon, e.g., the high book-to-market characteristic, and $1 - f$ that the investor follows a strategy with a low book-to-market characteristic.

Depending on their choices, each investor obtains gross returns X_a or X_b. We assume that investors with higher realized returns are emulated more often in their behavior by other investors than investors with low realized returns. This is payoff-biased transmission, a common assumption in the literature on cultural evolution.[16] One reason that this may occur is that investors who experience

[13] Other examples of agent-based models include LeBaron (2000, 2001, 2006), Hommes and Wagener (2009), Chiarella et al. (2009), and Lux (2009). See Lux and Zwinkels (2018) and Dieci and He (2018) for computational aspects of agent-based models.

[14] Barberis et al. (1998) consider agents who learn over a class of incorrect models about earnings, which generates under- and overreaction to earnings news. Hong et al. (2007) develop a model for learning in a multinomial world in which investors adapt to information on failing models. We model social contagion rather than belief learning, so the underlying mechanism that generates the price dynamics is different.

[15] Chinco (2023) develops a model for the *ex ante* likelihood of bubble based on the intensity of social interactions between speculators. Pedersen (2022) studies the GameStop event and shows how social network spillovers can explain influencers, thought leaders, momentum, reversal, bubbles, volatility, and volume.

[16] The mechanism that agents replicate based on past realized returns is also adopted by, for example, Lux (1995) and Brock and Hommes (1997, 1998). One subtle but important difference between our model and these classical models is that we deliberately avoid making assumptions about investor preferences over past returns. In fact, in our framework, the investor's preference itself can be determined by forces of evolution endogenously (Zhang et al., 2014).

high payoffs may tend to talk more about their returns with other investors, a phenomenon that Han et al. (2022) refer to as a self-enhancing transmission bias. Regardless of the channel, investors with higher realized returns will produce more offspring with the same philosophy (f) as themselves in the next period. We therefore make the following simple assumption.[17]

Assumption 3. X_a or X_b is also the number of offspring generated by the investment style a or b, respectively.

Hence, the number of offspring of individual i, X_i^f is given by:

$$X_i^f = I_i^f X_a + (1 - I_i^f) X_b, \quad I_i^f \equiv \begin{cases} 1 & \text{with probability } f \\ 0 & \text{with probability } 1 - f. \end{cases} \tag{1}$$

We assume that the trait value f is passed on without modification to newly infected individuals. As a result, the population may be viewed as being composed of "types" of individuals indexed by values of f that range from 0 to 1.

Equation (1) provides the model with the basis for its insights into the evolution of investor types over many generations, since it signifies the dynamics between periods. In focusing on the evolution of the distribution of types in the population, it differs from the large body of literature that instead focuses on the evolution of the distribution of wealth across investors.[18] The reproducing units in our framework are not investors or traders, but instead instances of investment philosophies. This has two implications.

First, one generation in our evolution model should not be interpreted as the actual lifetime of an investor, but rather the duration over which investment ideas spread. This can be weeks, days, or even minutes, as information spreads over social networks with modern information technology.

Second, our model can be modified to describe the switching of philosophies in a population of long-lived investors, as long as X_i^f is normalized by the total number of investors in the population. With this interpretation, investors' behaviors may depend on historical information beyond the returns in the current period. We provide such an example in Section 7.2; see also Lo and Zhang (2021) for an extension of the model in which agents have variable degrees of memory.

3.1. Population dynamics

Model generations are indexed by $t = 1, \cdots, T$ and investors in a given generation are indexed by i. We occasionally omit the subscript t since the randomness across time is IID. Finally, a superscript f denotes the particular type of investor as defined by the decision rule in (1).

Let n_t^f be the total number of type-f investors in period t, which is simply the sum of all the offspring from the type-f investors of the previous period:

$$n_t^f = \sum_{i=1}^{n_{t-1}^f} X_{i,t}^f = \left(\sum_{i=1}^{n_{t-1}^f} I_{i,t}^f \right) X_{at} + \left(\sum_{i=1}^{n_{t-1}^f} (1 - I_{i,t}^f) \right) X_{bt}. \tag{2}$$

Applying Kolmogorov's law of large numbers to $\sum_i I_{i,t}^f / n_{t-1}^f$ as n_{t-1}^f increases without bound, we derive the following almost sure population growth relationship between two periods:

$$n_t^f = n_{t-1}^f \left[f X_{at} + (1 - f) X_{bt} \right].$$

Through backward recursion, the population size of type-f investors in period T is

$$n_T^f = \prod_{t=1}^{T} \left[f X_{at} + (1 - f) X_{bt} \right] = \exp \left\{ \sum_{t=1}^{T} \log \left[f X_{at} + (1 - f) X_{bt} \right] \right\}, \tag{3}$$

where we have assumed that $n_0^f = 1$ without loss of generality. Taking the logarithm of the number of offspring, and once again applying Kolmogorov's law of large numbers, we have:

$$\frac{1}{T} \log n_T^f \xrightarrow{a.s.} \mathbb{E}[\log \left(f X_a + (1 - f) X_b \right)] \equiv \alpha(f) \tag{4}$$

as T increases without bound, where "$\xrightarrow{a.s.}$" in (4) denotes almost sure convergence. This is simply the expectation of the log-geometric average growth rate of the population, which we denote as $\alpha(f)$ henceforth. The optimal f that maximizes (4) is given by Brennan and Lo (2011):

[17] Some models assume a monotonic mapping from the gross returns X_a and X_b to the number of offspring (see Robson (1996), for example). Here we assume that this mapping is an identity function to simplify our analytical results. We generalize Assumption 3 in Section 5 and show that our results remain robust under a very general class of replication rules.

[18] Our approach is nevertheless broadly compatible with an interpretation based upon wealth accumulation. When investors with higher realized returns accumulate more wealth, they tend to have more resources, and therefore may become more influential in the population. This influence is directly analogous to spreading investment ideas to more individuals, which justifies the alternative perspective of Equation (1).

Proposition 1. *Under Assumptions 1–3, the growth-optimal type f^* that maximizes (4) is:*

$$f^* = \begin{cases} 1 & \text{if} \quad \mathbb{E}[X_b/X_a] < 1 \\ \text{solution to (6)} & \text{if} \quad \mathbb{E}[X_a/X_b] \geq 1 \quad \text{and} \quad \mathbb{E}[X_b/X_a] \geq 1 \\ 0 & \text{if} \quad \mathbb{E}[X_a/X_b] < 1, \end{cases} \tag{5}$$

where f^* is defined implicitly in the second case of (5) by:

$$\mathbb{E}\left[\frac{X_a - X_b}{f^* X_a + (1-f^*)X_b}\right] = 0 \tag{6}$$

and the expectations in (5)-(6) are with respect to the joint distribution $\Phi(X_a, X_b)$.

The growth-optimal type f^* is a function of the financial environment $\Phi(X_a, X_b)$. The role of Φ is critical in our framework, as it completely characterizes the effect of an investor's actions upon the type's reproductive success. The growth-optimal type f^* dominates the population in the long run because it grows exponentially faster than any other type. We will refer to f^* as the evolutionary equilibrium philosophy. It emerges through the forces of natural selection quite differently from the neoclassical economic framework of expected utility optimization.[19] in the evolutionary biology literature (Cooper and Kaplan, 1982; Frank and Slatkin, 1990; Frank, 2011).[20]

3.2. Style returns

Proposition 1 holds for any return distribution $\Phi(X_a, X_b)$ that satisfies Assumptions 1–2. However, it is interesting to give (X_a, X_b) a factor structure, and study how the contagion of investment ideas across investors affects the equilibrium investment philosophy f^*.

Let r be the common component of returns shared by styles a and b, ϵ_a and ϵ_b the style-specific components, and μ_a and μ_b the mean returns of styles a and b.

Assumption 4. The gross returns to the two styles are

$$X_a = \mu_a + \beta_a r + \epsilon_a$$
$$X_b = \mu_b + \beta_b r + \epsilon_b,$$

where $\beta_a > 0$ and $\beta_b > 0$ are the sensitivity of style returns to the common return component; r, ϵ_a and ϵ_b are independent and bounded random variables such that X_a and X_b are always positive; and $\mathbb{E}[r] = \mathbb{E}[\epsilon_a] = \mathbb{E}[\epsilon_b] = 0$.

Assumption 4 allows for a very wide set of possible investment styles. For instance, the two styles could be active versus passive investments, value versus growth stocks, fundamental versus quantitative strategies, domestic versus global investment, large firm versus small firm, long-only versus long-short, single-factor vs. multi-factor, and so forth. Different assumptions about the characteristics of μ_i, β_i, ϵ_i (where $i = a, b$), and r imply different cases of interest.

4. Evolutionary survival of investment styles

We next ask the question: how does the evolutionary equilibrium investment philosophy depend on the style return characteristics, including its expected returns, return betas, and return variances? We first identify the conditions for an equilibrium to consist solely of the choice of a single style, and then study the case where the long-run equilibrium population consists of investors who adopt both styles with positive probability. We briefly refer to empirical testing, but this topic is covered more extensively in Section 8 and Appendix E.

4.1. Single dominant style

By Proposition 1, the expected value of the ratios X_a/X_b and X_b/X_a determines whether the evolutionary equilibrium investment philosophy involves only one style, or a combination of the two. Let $y \equiv X_a/X_b$, so that

$$\mathbb{E}[y] = \mathbb{E}\left[\frac{X_a}{X_b}\right] = \mathbb{E}\left[\frac{\mu_a + \beta_a r + \epsilon_a}{\mu_b + \beta_b r + \epsilon_b}\right], \tag{7}$$

$$\mathbb{E}[1/y] = \mathbb{E}\left[\frac{X_b}{X_a}\right] = \mathbb{E}\left[\frac{\mu_b + \beta_b r + \epsilon_b}{\mu_a + \beta_a r + \epsilon_a}\right]. \tag{8}$$

[19] In fact, the evolutionary framework does not require a utility function initially, and the utility function itself can be endogenously determined by natural selection, as shown by Zhang et al. (2014).

[20] See Lo et al. (2021) for experimental evidence in the context of financial decision making.

For corner solutions, we focus on the case where style a dominates the population ($f^* = 1$). The case where style b dominates the population ($f^* = 0$) is similar. It is obvious from (8) that the following comparative statics on the conditions for $f^* = 1$ apply:

Proposition 2 *(Comparative Statics for Mean Return). Under Assumptions 1–4, style a-investors dominate the population if $\mathbb{E}[1/y] < 1$, which tends to occur ($\mathbb{E}[1/y]$ decreases) if:*

(i) *the mean return of style a, μ_a, increases;*
(ii) *the mean return of style b, μ_b, decreases.*

It is not surprising that a higher expected return of a style will promote its dominance in the population. To derive results for other return characteristics, we need to better understand $\mathbb{E}[y]$ and $\mathbb{E}[1/y]$. Applying the Taylor approximation of y as a function of r, ϵ_a and ϵ_b to estimate (7)-(8) we obtain

$$\begin{aligned} y(r,\epsilon_a,\epsilon_b) = \frac{X_a}{X_b} &= \frac{\mu_a + \beta_a r + \epsilon_a}{\mu_b + \beta_b r + \epsilon_b} \\ &= y(0,0,0) + \frac{\partial y_0}{\partial r} r + \frac{\partial y_0}{\partial \epsilon_a}\epsilon_a + \frac{\partial y_0}{\partial \epsilon_b}\epsilon_b \\ &\quad + \frac{1}{2}\left(\frac{\partial^2 y_0}{\partial r^2} r^2 + \frac{\partial^2 y_0}{\partial \epsilon_a^2}\epsilon_a^2 + \frac{\partial^2 y_0}{\partial \epsilon_b^2}\epsilon_b^2 + 2\frac{\partial^2 y_0}{\partial r \partial \epsilon_a} r\epsilon_a + 2\frac{\partial^2 y_0}{\partial r \partial \epsilon_b} r\epsilon_b + 2\frac{\partial^2 y_0}{\partial \epsilon_a \partial \epsilon_b}\epsilon_a\epsilon_b\right) + o(r^2, \epsilon_a^2, \epsilon_b^2). \end{aligned}$$

After taking the expected value of y, the linear terms vanish, since $\mathbb{E}[r] = \mathbb{E}[\epsilon_a] = \mathbb{E}[\epsilon_b] = 0$. The second-order cross terms also vanish because r, ϵ_a and ϵ_b are independent. Therefore, $\mathbb{E}[y]$ can be approximated by $y(0,0,0)$ and the second-order terms $Var(r)$, $Var(\epsilon_a)$ and $Var(\epsilon_b)$. A similar approximation applies for $\mathbb{E}[1/y]$, which is summarized in the following:

Lemma 1. *Under Assumptions 1–4, the second-order Taylor approximation with respect to r, ϵ_a and ϵ_b is*

$$\mathbb{E}[y] = \mathbb{E}\left[\frac{X_a}{X_b}\right] \approx \frac{\mu_a}{\mu_b} + \frac{\beta_a\beta_b^2}{\mu_b^3}\left(\frac{\mu_a}{\beta_a} - \frac{\mu_b}{\beta_b}\right) Var(r) + \frac{\mu_a}{\mu_b^3} Var(\epsilon_b),$$

$$\mathbb{E}[1/y] = \mathbb{E}\left[\frac{X_b}{X_a}\right] \approx \frac{\mu_b}{\mu_a} + \frac{\beta_a^2\beta_b}{\mu_a^3}\left(\frac{\mu_b}{\beta_b} - \frac{\mu_a}{\beta_a}\right) Var(r) + \frac{\mu_b}{\mu_a^3} Var(\epsilon_a).$$

We define μ_a/β_a and μ_b/β_b as a style's *scaled alpha*, which plays a critical role in determining the comparative statics for return beta and volatility, as shown in the next two propositions.

The scaled alpha has an interesting analogy to the slope of the security market line in the Capital Asset Pricing Model. In that model, all investor portfolios satisfy the same security market line slope, $(\overline{R} - R_F)/\beta$, where $\overline{R}$ is the investor's mean (net) return, R_F is the risk-free rate of return, and β is the portfolio's sensitivity to the return on the market. In our model, μ_a and μ_b are gross returns, and the scaled alpha can be decomposed into

$$\frac{\mu}{\beta} = \frac{1 + \overline{R}}{\beta} = \frac{\overline{R} - R_F}{\beta} + \frac{1 + R_F}{\beta}.$$

Therefore, if CAPM holds, the scaled alpha for the two investment styles differs only by $(1 + R_F)/\beta$. In the same market where R_F is a constant, this is determined by a style's beta, so that beta becomes the key determinant of strategy survival. The importance of scaled alpha will become clear after the following results.

Proposition 3 *(Comparative Statics for Return Beta). Under Assumptions 1–4, style a-investors dominate the population if $\mathbb{E}[1/y] < 1$. Up to a second-order Taylor approximation with respect to r, ϵ_a and ϵ_b, this tends to occur (that is, $\mathbb{E}[1/y]$ decreases) if:*

(i) *the sensitivity of style b to the common component, β_b, increases;*
(ii) *the sensitivity of style a to the common component, β_a, increases, conditional on style a's scaled alpha being sufficiently greater than style b's scaled alpha:*

$$\frac{\mu_a/\beta_a}{\mu_b/\beta_b} > 2;$$

(iii) *the sensitivity of style a to the common component, β_a, decreases, conditional on style a's scaled alpha being sufficiently small relative to style b's scaled alpha:*

$$\frac{\mu_a/\beta_a}{\mu_b/\beta_b} < 2.$$

The conditions for style a to dominate in the population are not symmetric with respect to β_a and β_b. First of all, a higher β_b will always promote the dominance of style a. Intuitively, this is because the log-geometric average growth rate in Equation (4) is nonlinear with respect to returns, and therefore the upside and downside for style b's realized returns do not offset. As a result, the high systematic risk of the competing style b promotes the success of style a because the risk causes near-extinctions of style b in the market selection process.

However, this is not always the case for β_a. For the same reason as described above, the high systematic risk of style a reduces its own success, but this is only true conditionally on style a's scaled alpha being comparable to or smaller than style b's. If the reverse is true, that is, if the mean return on style a is sufficiently strong relative to its risk (if style a's scaled alpha is sufficiently higher than style b's), the higher β_a actually encourages the dominance of style a in the population. In other words, style a's high scaled alpha serves as protection from its own downside risk.

We provide intuitions behind the asymmetry between β_a and β_b. Although styles a and b are symmetric in our model, Proposition 3 provides conditions under which *style a-investors* tend to dominate. From their perspective, the two styles are not symmetric because β_a represents the influence from its *own* beta while β_b represents the influence from the *other* style's beta. Mathematically, this reflects the nonlinearity in boundary conditions in Equations (7)–(8). In fact, if one considers conditions under which style b-investors tend to dominate, all results will be symmetric relative to Proposition 3, by replacing styles a with style b.

Proposition 4 *(Comparative Statics for Return Variance). Under Assumptions 1–4, style a-investors dominate the population if $\mathbb{E}[1/y] < 1$. Up to a second-order Taylor approximation with respect to r, ϵ_a and ϵ_b, this tends to occur (that is, $\mathbb{E}[1/y]$ decreases) if:*

(i) *the variance of style-specific component for a, $Var(\epsilon_a)$, decreases;*
(ii) *the variance of the common component, $Var(r)$, increases, conditional on style a's scaled alpha being greater than style b's scaled alpha:*

$$\frac{\mu_a}{\beta_a} > \frac{\mu_b}{\beta_b}$$

(iii) *the variance of the common component, $Var(r)$, decreases, conditional on style a's scaled alpha being smaller than style b's scaled alpha:*

$$\frac{\mu_a}{\beta_a} < \frac{\mu_b}{\beta_b}$$

Investment style a tends to dominate if its idiosyncratic variance is small, for essentially the same reason discussed earlier for return betas. A high variance tends to work against a style because of the nonlinearity of the long-term growth, as reflected in Equation (4); the upside and downside for style a's realized returns fail to offset.

Again, since we are considering the conditions for style a to dominate in this case, the results are not symmetric with respect to the idiosyncratic variances of style a and style b. It is interesting that style b's idiosyncratic variance does not affect style a's dominance (up to a second-order Taylor approximation).

The directional dependence on the variance of the common component is determined by the scaled alpha. A higher variance of the common component encourages style a to be dominant only if its scaled alpha is higher than style b's. Intuitively, a higher $Var(r)$ increases the variance of both investment styles, and the overall effect therefore depends on the relative sizes of the betas of both styles. However, the effect of risk also depends on the mean return. A high mean return acts as a buffer that reduces the importance of risk. It is therefore the scaled alpha that matters, not merely beta.

Propositions 2–4 together give a complete picture of the comparative effects on the conditions of $f^* = 1$ (that is, always choosing style a) for mean returns, return betas, and return variances. Parallel results can also be derived for $f^* = 0$ (always choosing style b) using approximations for $\mathbb{E}[y]$ in Lemma 1 instead. In the next section, we discuss mixed survival of investment styles.

4.2. The evolution of diversity

In general, if the evolutionary equilibrium philosophy involves both investment styles, f^* is given by (6). With Assumption 4, the first-order condition becomes:

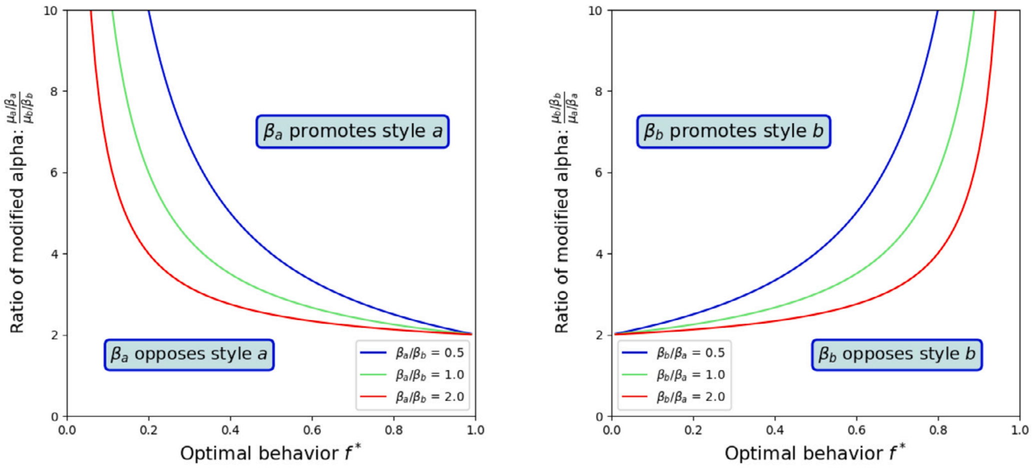
$$\mathbb{E}\left[\frac{(\mu_a - \mu_b) + (\beta_a - \beta_b)r + \epsilon_a - \epsilon_b}{[f\mu_a + (1-f)\mu_b] + [f\beta_a + (1-f)\beta_b]r + [f\epsilon_a + (1-f)\epsilon_b]}\right] = 0. \tag{9}$$

Taking derivatives of Equation (9) to μ_a and μ_b, we immediately have the following comparative statics for the philosophy f^*.

Proposition 5 *(Comparative Statics for Mean Return). Under Assumptions 1–4, when the evolutionary equilibrium philosophy has mixed investment styles, the equilibrium philosophy f^* increases when:*

(i) *the mean return of style a, μ_a, increases;*
(ii) *the mean return of style b, μ_b, decreases.*

Not surprisingly, Proposition 5 is similar to Proposition 2; they both assert that a higher expected return encourages investment in that style. To empirically test Propositions 2 and 5, one can estimate historical mean returns of value versus growth stocks, and see if a change in their realized returns over time corresponds to change in the frequencies of value versus growth investors. These can be estimated, e.g., from mutual fund holdings or social media data. In the context of hedge funds, one can look at the average



(a) Comparative statics for β_a (b) Comparative statics for β_b

Fig. 1. Comparative Statics for Return Beta: β_a (1a) and β_b (1b). In the case of β_a (1a), the vertical axis represents the ratio of the scaled alpha, $\frac{\mu_a/\beta_a}{\mu_b/\beta_b}$, and the horizontal axis represents the evolutionary equilibrium philosophy, f^*. Three lines of different colors represent the boundaries between promoting and demoting style a for three different ratios of beta, β_a/β_b. The upper region represents when β_a promotes style a, while the lower region represents when β_a opposes style a. The case of β_b (1b) is symmetrical.

return of different investment styles, such as fundamental versus quantitative in a certain period, and correlate that with attrition rates in different hedge fund categories. Section 8 and Appendix E discuss directions for empirical tests in more detail.

To derive further comparative statics, we again use a Taylor expansion to approximate the first-order condition of Equation (9).

Lemma 2. *Under Assumptions 1–4, up to a second-order Taylor approximation with respect to r, ϵ_a and ϵ_b, the first-order condition (9) is*

$$0 = (\mu_a - \mu_b)\left[f\mu_a + (1-f)\mu_b\right]^2 + \beta_a\beta_b\left[f\beta_a + (1-f)\beta_b\right]\left(\frac{\mu_a}{\beta_a} - \frac{\mu_b}{\beta_b}\right)Var(r)$$
$$+ (1-f)\mu_a Var(\epsilon_b) - f\mu_b Var(\epsilon_a).$$

When $\mathbb{E}[X_a/X_b] \geq 1$ and $\mathbb{E}[X_b/X_a] \geq 1$, the evolutionary equilibrium philosophy involves mixed investment styles, and f^* is given by Lemma 2, up to a second-order Taylor approximation.

Proposition 6 *(Comparative Statics for Return Beta). Under Assumptions 1–4, when the evolutionary equilibrium philosophy has mixed investment styles, up to a second-order Taylor approximation with respect to r, ϵ_a and ϵ_b, the equilibrium philosophy f^* increases when:*

(i) *β_a increases, if* $\frac{\mu_a/\beta_a}{\mu_b/\beta_b} > 2 + \frac{1-f^*}{f^*}\left(\frac{\beta_b}{\beta_a}\right)$;
(ii) *β_a decreases, if* $\frac{\mu_a/\beta_a}{\mu_b/\beta_b} < 2 + \frac{1-f^*}{f^*}\left(\frac{\beta_b}{\beta_a}\right)$[21];
(iii) *β_b decreases, if* $\frac{\mu_b/\beta_b}{\mu_a/\beta_a} > 2 + \frac{f^*}{1-f^*}\left(\frac{\beta_a}{\beta_b}\right)$;
(iv) *β_b increases, if* $\frac{\mu_b/\beta_b}{\mu_a/\beta_a} < 2 + \frac{f^*}{1-f^*}\left(\frac{\beta_a}{\beta_b}\right)$.[22]

The relationship between the evolutionary equilibrium philosophy f^* and return beta is determined by three components: the ratio of scaled alphas, the ratio of betas, and the philosophy f^*. Fig. 1 shows the regions in which the return beta promotes or opposes the investment style, as a function of the ratio of the scaled alpha and the philosophy f^*.

Proposition 6 generalizes Proposition 3 from the case of a single dominant style to the case of mixed styles. To see this, suppose style a is dominant and $f^* = 1$. The condition in the fourth item of Proposition 6 is always true, and therefore the dominance tends to occur when β_b increases, which corresponds to the first item of Proposition 3, and $f^* = 1$ in Fig. 1b. Similarly, the condition in the first two items of Proposition 6 reduces to the second and third item of Proposition 3 trivially, and this corresponds to $f^* = 1$ in Fig. 1a. Once again, the scaled alphas μ_a/β_a and μ_b/β_b play a critical role in determining the direction of beta's impact on the philosophy f^*. Instead of threshold 2 in Proposition 3, the threshold here is adjusted by a positive amount, the adjustment depending on f^* and β_a/β_b, as shown in Fig. 1.

[21] This is always true when $f^* \leq \frac{\mu_b}{\mu_a+\mu_b}$, since the right-hand side reduces to $2 + \frac{\mu_a/\beta_a}{\mu_b/\beta_b}$.
[22] This is always true when $f^* \geq \frac{\mu_b}{\mu_a+\mu_b}$, since the right-hand side reduces to $2 + \frac{\mu_b/\beta_b}{\mu_a/\beta_a}$.

Factor sensitivity β_a always opposes style a when $f^* \leq \frac{\mu_b}{\mu_a+\mu_b}$. Intuitively, this means that when the equilibrium frequency of style a-investors is small relative to the proportion of style b's expected return $\frac{\mu_b}{\mu_a+\mu_b}$, it promotes the survival of a philosophy to decrease the weight of style a as style a's beta increases. On the other hand, when $f^* \geq \frac{\mu_b}{\mu_a+\mu_b}$, it promotes the survival of a philosophy to decrease the weight of style b as style b's beta increases, symmetric to the case for β_a.

When two investment styles have comparable scaled alphas ($\frac{\mu_a/\beta_a}{\mu_b/\beta_b} \approx 1$), β_a opposes style a and β_b opposes style b. In other words, a lower beta investment style is preferred if its scaled alpha is comparable to other styles in the market. In the context of hedge funds, a testable implication is that a low beta strategy should attract more investors after controlling for factors such as expected return and volatility, especially when the scaled alpha is comparable with alternative investment styles.

This result is derived using exogenous returns (Assumption 4). However, if investors are attracted to the low-beta style, they may drive up its price and drive down its expected return, which tends to have a negative feedback effect on the survival of the low-beta style. Nevertheless, we show that these results hold in a market equilibrium setting with endogenous returns in Section 6.

In contrast, if one investment style has a much higher scaled alpha than the other style (corresponding to the upper regions in Fig. 1), a higher beta actually promotes the popularity of that style. This is because the scaled alpha is so large that it gives substantial downside protection against any increase in variance brought by a higher beta. More variance becomes good for survival in this case. For alternative investments such as hedge funds, private equity and venture capital, the expected return can be very high and the beta can be very low. Therefore, the scaled alpha for these investments can be much higher than that for traditional investment styles. Our model predicts that high-beta styles are favored in this case. In the context of the stock market, this implies that investment styles in high beta stocks will gain popularity if their scaled alphas are sufficiently high, leading to a decrease in returns. In contrast, investment styles in low beta stocks lose popularity, leading to higher returns. This outcome is consistent with the empirical anomaly that low beta stocks earn high expected returns, as contrasted with the traditional risk premium theory that they should earn low expected returns. Our result can therefore justify the use of a common defensive (low-risk) "smart beta" strategy (Frazzini and Pedersen, 2014).

Proposition 7 *(Comparative Statics for Return Variance). Under Assumptions 1–4, when the evolutionary equilibrium philosophy has mixed investment styles, up to a second-order Taylor approximation with respect to r, ϵ_a and ϵ_b, the equilibrium philosophy f^* increases when:*

(i) *the variance of style-specific component for a, $Var(\epsilon_a)$, decreases;*
(ii) *the variance of style-specific component for b, $Var(\epsilon_b)$, increases;*
(iii) *the variance of the common component, $Var(r)$, increases, conditional on style a's scaled alpha being greater than style b's scaled alpha: $\frac{\mu_a}{\beta_a} > \frac{\mu_b}{\beta_b}$;*
(iv) *the variance of the common component, $Var(r)$, decreases, conditional on style a's scaled alpha being smaller than style b's scaled alpha: $\frac{\mu_a}{\beta_a} < \frac{\mu_b}{\beta_b}$.*

There will be more style a-investors if style a's idiosyncratic variance is small, and if style b's idiosyncratic variance is large. This also generalizes Proposition 4 from the case of a single dominant style to the case of mixed styles. Intuitively, a higher style-specific variance discourages investment in that style,[23] because of the nonlinearity of the long-term growth as reflected in Equation (4). In other words, the possibility of near wipe-outs of an investment style is disproportionately important, opposing the survival of more volatile investment styles.

The directional dependence of the equilibrium philosophy on the variance of the common component is again determined by the scaled alpha. A higher variance of the common component encourages investment in the style with a higher scaled alpha. The reason is similar to that in our previous discussions. A higher $Var(r)$ increases the variance of both investment styles, and the overall effect therefore depends on the relative sizes of the betas of both styles. However, the effect of risk also depends on the mean return. A high mean return acts as a buffer that reduces the importance of risk. It is therefore the scaled alpha that matters, not only a comparison of betas.

Proposition 4 and 7 offer interesting new possibilities for the empirical consequences of return variance. In the context of hedge funds, one can test whether high idiosyncratic variance in returns opposes the survival of that investment style, or even specific fund managers with allocations in that style. Industry practitioners often use the Sharpe ratio to select fund managers. If hedge funds truly deliver returns with low correlation to the broader markets, a high Sharpe ratio would directly correspond to low idiosyncratic return variance, consistent with the implications of our model.

Moreover, the effect of the variance of the common component depends on each strategy's scaled alpha. The variance of the common component of two investment styles in general corresponds to the volatility of broader factors such as the market portfolio. This implies that during volatile times, investors with higher scaled alpha tend to flourish. This is directly testable in both individual investment strategies and hedge funds. For example, one can compare the frequency of investors in value versus growth strategies, momentum versus defensive, and so on, during periods of high and low market volatility, and test whether high market volatility promotes survival of those types that invest heavily in styles with high scaled alpha. With hedge fund data, one can study the

[23] In Han et al. (2022) the opposite is true: variance promotes survival. In Han et al. (2022), this effect is driven by a selection bias whereby high returns are more likely to be reported, which is intensified by high variances. The model here allows for a more general distribution in the number of offspring, which results in a distinct intertemporal dynamic effect: a long-run "evolutionary hedging" benefit to avoid very low reproduction outcomes.

attrition rates of different investment styles through different market cycles, testing the similar hypothesis that high market volatility promotes hedge fund categories with high scaled alpha.

We emphasize that our comparative statics results with respect to beta and volatility hold true up to second-order Taylor approximations.[24] These approximations are for y and $1/y$ in (7)–(8), and ultimately their expected values determine the survival philosophy. As a result, second-order Taylor approximations allow us to derive analytical insights up to the second moment of returns, such as volatility, but they do not account for higher-order moments, such as the skewness of returns, which are left for future studies.

We apply these results to study an example of special return properties in Appendix A.

5. General replication rules

Our basic model has assumed that the replication rule, i.e., the mapping from the returns to the number of offspring, is an identity function (see Assumption 3). Here we consider a general class of replication rules and assess the robustness of the results of our derivations so far.

5.1. General replication function and equilibrium philosophy

We first generalize Assumption 3 to allow for a much more general class of replication rules.

Assumption 5. The number of offspring generated by the investment style a or b is given by $\psi(X_a)$ or $\psi(X_b)$, where $\psi(\cdot)$ is a *replication function* that is twice differentiable, non-negative, and non-decreasing:

$$\psi \geq 0 \quad \text{and} \quad \psi' \geq 0.$$

Assumption 6. The replication function is concave: $\psi'' \leq 0$.

Assumption 5 reflects a few natural conditions for any reasonable evolutionary process. $\psi \geq 0$ guarantees that the number of offspring is non-negative. $\psi' \geq 0$ guarantees that higher returns are preferred and therefore do not lead to fewer followers. In Assumption 6, $\psi'' \leq 0$ corresponds to a diminishing marginal effect of return-biased transmission, that is, an increase in returns from 1% to 2% will be more influential than that from 10% to 11%. However, Assumption 6 may not be true for all markets. For example, lottery markets have low expected returns, yet they persistently attract investors. Lottery-like preferences imply that extreme returns attract an overwhelming amount of attention and investments, which is reflected by a convex replication function with $\psi'' > 0$.[25]

By following the same derivations as in Equations (2)–(4), it is easy to show that the average log population for philosophy f satisfies:

$$\frac{1}{T}\log n_T^f \xrightarrow{a.s.} \mathbb{E}[\log\left(f\psi(X_a)+(1-f)\psi(X_b)\right)] \equiv \alpha_\psi(f) \tag{10}$$

as T increases without bound. We add the subscript "$_\psi$" to the population growth rate $\alpha_\psi(f)$, which emphasizes the fact that ψ determines the growth rate, and therefore, the optimal investment philosophy. The optimal f that maximizes (10) is given by:

Proposition 8. *Under Assumptions 1, 2, and 5, the growth-optimal type f_ψ^* that maximizes (10) is:*

$$f_\psi^* = \begin{cases} 1 & \text{if} \quad \mathbb{E}\left[\psi(X_b)/\psi(X_a)\right] < 1 \\ \text{solution to (12)} & \text{if} \quad \mathbb{E}\left[\psi(X_a)/\psi(X_b)\right] \geq 1 \quad \text{and} \quad \mathbb{E}\left[\psi(X_b)/\psi(X_a)\right] \geq 1 \\ 0 & \text{if} \quad \mathbb{E}\left[\psi(X_a)/\psi(X_b)\right] < 1, \end{cases} \tag{11}$$

where f_ψ^* is defined implicitly in the second case of (11) by:

$$\mathbb{E}\left[\frac{\psi(X_a)-\psi(X_b)}{f_\psi^*\psi(X_a)+(1-f_\psi^*)\psi(X_b)}\right] = 0 \tag{12}$$

and the expectations in (11)-(12) are with respect to the joint distribution $\Phi(X_a, X_b)$.

We can derive a parallel set of comparative statics for f_ψ^* with respect to return characteristics. In general, the results in Propositions 2–7 are robust to general replication functions ψ, although in certain cases, explicit characterizations of boundary conditions are no longer possible in terms of simple expressions of μ and β. We summarize the key conclusions here, and leave the mathematical details to the proofs in Appendix F.

[24] Results with respect to mean returns do not rely on Taylor approximations.

[25] Or at least convex when returns X_a and X_b are high.

5.2. Single dominant style

We first provide the results for comparative statics when f_ψ^* is either 0 or 1.

Proposition 9 *(Single Dominant Rules under General Replication Functions). Under Assumptions 1, 2, 4, 5, and 6, style a-investors tend to dominate the population if:*

(i) (a) *the mean return of style a, μ_a, increases;*
 (b) *the mean return of style b, μ_b, decreases.*

(ii) (a) *β_b increases;*
 (b) *β_a increases, conditional on style a's ψ-scaled alpha being sufficiently greater than style b's ψ-scaled alpha:*

$$\frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b} > C_1;$$

 (c) *β_a decreases, conditional on style a's ψ-scaled alpha being sufficiently small relative to style b's ψ-scaled alpha:*

$$\frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b} < C_1;$$

(iii) (a) *the variance of the style-specific component for a, $Var(\epsilon_a)$, decreases;*
 (b) *the variance of the common component, $Var(r)$, increases, conditional on style a's ψ-scaled alpha being sufficiently greater than style b's ψ-scaled alpha:*

$$\frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b} > C_2;$$

 (c) *the variance of the common component, $Var(r)$, decreases, conditional on style a's ψ-scaled alpha being sufficiently small relative to style b's ψ-scaled alpha:*

$$\frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b} < C_2.$$

The conditions in (ii)–(iii) hold to a second-order Taylor approximation with respect to r, ϵ_a and ϵ_b. C_1 and C_2 are given in the proof in Appendix F.

In Proposition 9, case (i) generalizes Proposition 2, case (ii) generalizes Proposition 3, and case (iii) generalizes Proposition 4.

Next, we relax Assumption 6 to allow for lottery-like preferences. In this case, higher style-specific variances actually promote survival.[26]

Proposition 10 *(Single Dominant Rules under Lottery-like Replication Functions). Under Assumptions 1, 2, 4, and 5, when there exist lottery-like preferences such that the replication function is convex: $\psi'' > 0$, style a-investors tend to dominate the population if:*

(i) *the mean return of style a, μ_a, increases;*
(ii) *the variance of style-specific component for a, $Var(\epsilon_a)$, increases;*

where (ii) holds to a second-order Taylor approximation with respect to r, ϵ_a and ϵ_b.

5.3. Diverse investment styles

The next result provides the comparative statics when the evolutionary equilibrium philosophy includes both investment styles.

Proposition 11 *(Diversity under General Replication Functions). Under Assumptions 1, 2, 4, 5, and 6, when the evolutionary equilibrium philosophy has mixed investment styles, the equilibrium philosophy f^* increases when:*

(i) (a) *the mean return of style a, μ_a, increases;*
 (b) *the mean return of style b, μ_b, decreases.*

(ii) (a) *β_a increases, if $\frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b} > C_3$;*
 (b) *β_a decreases, if $\frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b} < C_3$;*
 (c) *β_b decreases, if $\frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b} > C_3'$;*
 (d) *β_b increases, if $\frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b} < C_3'$.*

[26] We thank an anonymous reviewer for suggesting this idea.

(iii) (a) *the variance of the style-specific component for* a, $Var(\epsilon_a)$, *decreases;*
(b) *the variance of the style-specific component for* b, $Var(\epsilon_b)$, *increases;*
(c) *the variance of the common component,* $Var(r)$, *increases, conditional on style* a*'s* ψ*-scaled alpha being sufficiently greater than style* b*'s* ψ*-scaled alpha:*

$$\frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b} > C_4;$$

(d) *the variance of the common component,* $Var(r)$, *decreases, conditional on style* a*'s* ψ*-scaled alpha being sufficiently small relative to style* b*'s* ψ*-scaled alpha:*

$$\frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b} < C_4.$$

The conditions in (ii)–(iii) hold to a second-order Taylor approximation with respect to r, ϵ_a *and* ϵ_b. C_3, C_3', *and* C_4 *are given in the proof in Appendix F.*

In Proposition 11, case (i) generalizes Proposition 5, case (ii) generalizes Proposition 6, and case (iii) generalizes Proposition 7.

Like the case of singly dominant rules, when the replication function reflects lottery-like preferences, higher style-specific variances promote survival.

Proposition 12 *(Diversity under Lottery-Like Replication Functions). Under Assumptions 1, 2, 4, and 5, when there exist lottery-like preferences such that the replication function is sufficiently convex:* $\psi'' > C_5$ *where* $C_5 > 0$ *is given in the proof in Appendix F, and when the evolutionary equilibrium philosophy has mixed investment styles, the equilibrium philosophy* f^* *increases if:*

(i) *the mean return of style* a, μ_a, *increases;*
(ii) *the mean return of style* b, μ_b, *decreases;*
(iii) *the variance of the style-specific component for* a, $Var(\epsilon_a)$, *increases;*
(iv) *the variance of the style-specific component for* b, $Var(\epsilon_b)$, *decreases;*

where (iii)–(iv) hold to a second-order Taylor approximation with respect to r, ϵ_a *and* ϵ_b.

Overall, Propositions 9–12 show that the equilibrium philosophy f_ψ^* has the same set of dependencies on return characteristics as those for f^*, with only a different notion of ψ-scaled alpha and a different set of constants specifying its boundary conditions. These results confirm that our key conclusions in Section 4 are robust to a very general class of replication rules.

In addition, by considering general replication rules that are convex, our model provides an explanation for the persistence of lottery markets and lottery-like stocks despite their excess risk, and therefore lower risk-adjusted expected returns. This effect is also consistent with Han et al.'s (2022) model of social transmission bias.

6. Diversity in market equilibrium

So far, we have viewed the returns on investment style as exogenous (Assumption 4). However, in market equilibrium, stock returns reflect shifts in supply and demand as the frequencies of different investment styles shift. In this section, we extend the model to reflect the fact that the imbalance between supply and demand for the securities traded by styles a and b affects their expected returns. In particular, we build an equilibrium model with endogenous returns and study its implications for the equilibrium investment philosophy.

6.1. An equilibrium model

Fundamental value vs actual price We start by making a distinction between the fundamental value and the actual price of style a and b. We interpret X_{at} and X_{bt} defined in Assumption 4 as gross returns to the *fundamental value* processes of style a and b. We use $\tilde{P}_{a,t}$ and $\tilde{P}_{b,t}$ to denote the corresponding fundamental values, which are given by:

$$\begin{aligned} \tilde{P}_{a,t} &= X_{at} \cdot \tilde{P}_{a,t-1}, \\ \tilde{P}_{b,t} &= X_{bt} \cdot \tilde{P}_{b,t-1}. \end{aligned} \tag{13}$$

On the other hand, the *actual prices*, $P_{a,t}$ and $P_{b,t}$, may deviate from fundamental values due to forces of supply and demand in the market. They are related to the actual returns, R_{at} and R_{bt}, by:

$$\begin{aligned} P_{a,t} &= R_{at} \cdot P_{a,t-1}, \\ P_{b,t} &= R_{bt} \cdot P_{b,t-1}. \end{aligned} \tag{14}$$

The distinction between fundamental value and actual price follows the classical work of Lux (1995), who built a model of herd behavior in speculative markets in which the demand for a single asset is determined by deviations of its price from the fundamental value.[27]

Demand, supply, and market clearing Let $U = \left\{0, \frac{1}{K}, \frac{2}{K}, \cdots, 1\right\} = \{f_1, f_2, \cdots, f_{K+1}\}$ be a discrete universe that consists of $K+1$ types of investors. Let q_t^f be the frequency of type-f investors in the population in period t:

$$q_t^f = \frac{n_t^f}{\sum_{g\in U} n_t^g}, \tag{15}$$

so that the frequencies of all types of investors sum to one. Let the *aggregate demand* in style a in period t be the frequency-weighted average investment philosophy in the population:

$$\lambda_t = \sum_{f\in U} f q_t^f. \tag{16}$$

By definition, the aggregate demand begins at 0.5, and evolves to a value between zero and one as the two investment styles generate different returns.

Following the literature on heterogeneous agent models and noise traders (Lux, 1995, 2009; Lux and Marchesi, 2000; Farmer and Joshi, 2002; Chiarella et al., 2009; Hommes and Wagener, 2009), we make a distinction between *speculators* and *fundamentalists* in the market. Speculators refer to investors we have considered so far, and their intertemporal dynamics follow the return-biased transmission. Given actual prices, the dollar demand from speculators in the market for asset a and asset b are $W_S\lambda_t$ and $W_S(1-\lambda_t)$, where W_S is the total wealth of all speculators. Therefore, the demand in shares is:

$$D_{a,t} = \frac{W_S\lambda_t}{P_{a,t}}, \qquad D_{b,t} = \frac{W_S(1-\lambda_t)}{P_{b,t}}. \tag{17}$$

On the other hand, a second group of traders, the fundamentalists, offer supply in the market. Their supply is determined by the difference between the fundamental value and actual price[28]:

$$S_{a,t} = \frac{W_F\left(P_{a,t}/\tilde{P}_{a,t}\right)^k}{P_{a,t}}, \qquad S_{b,t} = \frac{W_F\left(P_{b,t}/\tilde{P}_{b,t}\right)^k}{P_{b,t}}, \tag{18}$$

where W_F represents the total wealth from the fundamentalists, $\left(P_{a,t}/\tilde{P}_{a,t}\right)^k$ and $\left(P_{b,t}/\tilde{P}_{b,t}\right)^k$ represent the dollar supply for asset a and b, and $k>0$ is a constant measuring the elasticity or sensitivity of supply with respect to deviations from the fundamental value. A larger k corresponds to a higher sensitivity from fundamentalists in response to such deviations.[29]

When the market clears, supply must equal demand. Therefore we have:

$$\begin{aligned} D_{a,t} = S_{a,t} &\implies \frac{W_S\lambda_t}{P_{a,t}} = \frac{W_F\left(P_{a,t}/\tilde{P}_{a,t}\right)^k}{P_{a,t}}, \\ D_{b,t} = S_{b,t} &\implies \frac{W_S(1-\lambda_t)}{P_{b,t}} = \frac{W_F\left(P_{b,t}/\tilde{P}_{b,t}\right)^k}{P_{b,t}}. \end{aligned} \tag{19}$$

Price fluctuations are caused by the endogenous mechanism relating the fraction of investors choosing style a to the distance between the fundamental value and actual price.[30]

[27] It is worth noting that many models in this literature start with an exogenous stochastic dividend process, and solve for equilibrium prices given a certain class of beliefs or investment strategies. See, for example, Brock and Hommes (1997, 1998), Hong et al. (2007), Evstigneev et al. (2006, 2008), Bottazzi and Dindo (2014), Bottazzi et al. (2018). In our model, we are particularly interested in the relationship between survival and asset return characteristics such as mean returns, betas, systematic risk, and idiosyncratic volatility. Therefore, it is more appropriate in our case to start with a factor model of the value (return) rather than the cash flow (dividend).

[28] In this sense, the fundamentalists can also be regarded as market makers to meet the demand. An alternative interpretation is that the fundamentalists are also generating demand in the market together with the speculators. The aggregate demand is met by a constant one unit of supply, the specification used by Lux (1995) which is equivalent to our specification here.

[29] We choose the specification in Equation (18) because it allows us to derive analytical results explicitly. Alternative specifications are possible as long as the supply depends on deviations from the fundamental value.

[30] The relative wealth between the speculators and fundamentalists, W_S and W_F, can also be modeled, and one can study the survival of speculators (noise traders) versus fundamentalists, and their impact on asset prices. However, the survival of noise traders has been extensively studied and is not the focus of our paper (see, for example, De Long et al. (1990, 1991), Kyle and Wang (1997), Hirshleifer and Luo (2001), Hirshleifer et al. (2006), Yan (2008), Kogan et al. (2006, 2017)). We take a simpler route, and hold the fraction of speculators versus fundamentalists constant, which is enough to model the dependence of endogenous prices on demand fluctuations for styles a and b.

6.2. Prices, returns, and philosophy in equilibrium

Equilibrium prices and returns Solving Equation (19) for market clearing conditions, we have the following result for equilibrium prices and returns.

Proposition 13. *In the market equilibrium model, the endogenous equilibrium prices are given by:*

$$\begin{aligned} P_{a,t} &= \tilde{P}_{a,t}\left(\frac{W_S \lambda_t}{W_F}\right)^{\frac{1}{k}}, \\ P_{b,t} &= \tilde{P}_{b,t}\left(\frac{W_S(1-\lambda_t)}{W_F}\right)^{\frac{1}{k}}, \end{aligned} \tag{20}$$

and the endogenous equilibrium returns are given by:

$$\begin{aligned} R_{a,t} &= X_{a,t}\left(\frac{\lambda_t}{\lambda_{t-1}}\right)^{\frac{1}{k}}, \\ R_{b,t} &= X_{b,t}\left(\frac{1-\lambda_t}{1-\lambda_{t-1}}\right)^{\frac{1}{k}}. \end{aligned} \tag{21}$$

There are several interesting observations that can be made from Proposition 13. First, the aggregate demand (λ_t) determines the equilibrium prices and their deviations from the fundamental value, while it is the *change* in aggregate demand between two periods (λ_t/λ_{t-1}) that determines the equilibrium returns. For example, as style a generates higher returns, investors with higher f will generate more offspring in the next period, driving the aggregate demand in style a higher. As a result, we expect the cost of purchasing style a securities to increase, which reduces the return for buying and holding style a.

Second, the equilibrium prices are affected by the fraction of speculators versus fundamentalists in the market (W_F/W_S). Because our model does not focus on how this fraction changes over time, the price dynamics are mainly driven by the relative demand (λ_t).

Third, the exponent $1/k$ describes the shape of a power-law market impact from trading, which is the reciprocal of the sensitivity to price deviations by the fundamentalists. Higher sensitivities lead to a milder price impact, and lower sensitivities lead to a stronger price impact. This is closely related to Kyle's (1985) market microstructure model in which liquidity is measured by an estimate of the log-volume required to move the price by one dollar.[31]

Finally, if we consider price deviations from the fundamental value:

$$\begin{aligned} \frac{P_{a,t}}{\tilde{P}_{a,t}} &= \left(\frac{W_S \lambda_t}{W_F}\right)^{\frac{1}{k}}, \\ \frac{P_{b,t}}{\tilde{P}_{b,t}} &= \left(\frac{W_S(1-\lambda_t)}{W_F}\right)^{\frac{1}{k}}, \end{aligned} \tag{22}$$

our model implies that higher demand in a style (λ_t) leads to a higher degree of price deviation, what might be considered a bubble, and a higher level of supply sensitivity (k) makes it more difficult to substantially deviate from the fundamental values, in other words, less likely to form bubbles.

Equilibrium philosophy with endogenous returns Given the endogenous returns in Proposition 13, we denote an equilibrium philosophy by f^e, with superscript e indicating endogenous returns.

Proposition 14. *Under Assumptions 1, 2, and 4 and the endogenous returns given by the market clearing conditions of Equations (19)–(21), the equilibrium philosophy f^e that maximizes the investor's growth as T increases without bound is identical to f^* in Proposition 1 under the simple replication rule given by Assumption 3, and to f^*_ψ in Proposition 8 under the general replication rule given by Assumptions 5–6.*

Proposition 14 shows that though asset prices are affected in the long run by the relative demand in style a to style b, the equilibrium philosophy remains the same. In other words, our results in Propositions 2–11 remain robust in a model of market equilibrium. This is not surprising given our remarks after Proposition 13. Indeed, equilibrium prices are affected by the aggregate demand in the long run. However, the equilibrium returns of Equation (21) are determined by two terms—the returns on the fundamental value, and an adjustment term that depends on the *change* in demand between two periods. In equilibrium, the second term vanishes to a constant one.

A large literature on market selection has documented that the survival of traders differs markedly from their price influence in the market (Kogan et al., 2006, 2017; Cvitanić and Malamud, 2011; Easley and Yang, 2015). Propositions 13–14 allow us to study whether different surviving philosophies in equilibrium necessarily imply different prices.

[31] See also Bertsimas and Lo (1998), Lillo et al. (2003), and Almgren et al. (2005) for more detailed explorations of the power law of price impact in equity markets.

We consider two scenarios with two different equilibrium philosophies, f_1^e and f_2^e, which imply $\lambda_{t,1}$ and $\lambda_{t,2}$, two equilibrium aggregate demands for style a. We follow Easley and Yang (2015) to consider the ratio of equilibrium prices normalized by fundamental values in these two scenarios, $\frac{P_{a,t,1}/\tilde{P}_{a,t,1}}{P_{a,t,2}/\tilde{P}_{a,t,2}}$ and $\frac{P_{b,t,1}/\tilde{P}_{b,t,1}}{P_{b,t,2}/\tilde{P}_{b,t,2}}$, where the subscripts 1 and 2 denote the two scenarios. Equation (20) implies that:

$$\begin{aligned}\frac{P_{a,t,1}/\tilde{P}_{a,t,1}}{P_{a,t,2}/\tilde{P}_{a,t,2}} &= \left(\frac{\lambda_{t,1}}{\lambda_{t,2}}\right)^{\frac{1}{k}} = \left(\frac{f_1^e}{f_2^e}\right)^{\frac{1}{k}},\\ \frac{P_{b,t,1}/\tilde{P}_{b,t,1}}{P_{b,t,2}/\tilde{P}_{b,t,2}} &= \left(\frac{1-\lambda_{t,1}}{1-\lambda_{t,2}}\right)^{\frac{1}{k}} = \left(\frac{1-f_1^e}{1-f_2^e}\right)^{\frac{1}{k}},\end{aligned} \tag{23}$$

where the right-hand side shows the difference between the two surviving philosophies, while the left-hand side shows the difference between the equilibrium prices.

This relationship shows that, although different philosophies may survive under different style return distributions, their influences on equilibrium prices are milder due to the concavity of the function in Equation (23) when $k > 1$. In our model, k represents the elasticity of supply with respect to price deviations from the fundamental value, and more competitive markets imply higher values of k. Table A.1 in Appendix B demonstrates this relationship for several different values of k. For example, when $k = 5$, a two-fold difference in equilibrium philosophies implies a price difference of only 15% at equilibrium.

This phenomenon is similar to that found in Easley and Yang (2015), who find that although market selection in terms of wealth share may be slow for different preferences—in their case, loss aversion versus arbitrageurs—the price impact from investors with loss aversion may be much smaller. We do not model preferences in our framework. Instead, preferences are implicitly reflected by how investors choose between the two investment styles, i.e., the philosophy f. Nonetheless, our model highlights a similar phenomenon that market selection in terms of the surviving philosophy and its price impact can be quite varied, especially in competitive markets where the elasticity of supply with respect to price deviations is high.

Appendix B provides two simulated examples to further demonstrate the effect of market equilibrium. In certain cases, market equilibrium in fact speeds up the rate of convergence.

7. Psychological bias and investment philosophy

We have assumed so far that investors are only influenced by the observed payoffs. In reality, investors may also be persuaded to adopt an investment philosophy based upon whether someone else has adopted it. In this section, we discuss two such psychological effects.[32]

7.1. Conformist preference

Investors may have conformist preferences (Klick and Parisi, 2008), perhaps through the mechanism of viewing other investors as being better informed, and therefore will be influenced by the choices of others. We generalize the population dynamics between two generations in Equation (1) to capture this effect:

$$X_{i,t}^f = \left[I_{i,t}^f X_{at} + (1 - I_{i,t}^f) X_{bt}\right] \exp\left[\tau(f - \lambda_{t-1})^2\right], \tag{24}$$

where λ_{t-1} is the average philosophy in the population in the previous generation $t-1$, and $\tau <= 0$ is the intensity of conformity pressure. When $\tau < 0$, the further f is away from the average philosophy λ_{t-1}, the more intense is the conformity pressure.

The magnitude of the conformity pressure τ acts roughly as a multiplicative factor in the fitness, or an additive factor in the population growth rate (see Appendix D.1). Suppose a long time has passed, and the evolutionary equilibrium philosophy f^* that maximizes $\alpha(f)$ without conformity pressure has dominated the population. The investment philosophy f^* is evolutionarily stable because any other philosophy grows even more slowly than f^* with a negative conformity pressure term. However, if f^* is not initially popular, it may never grow. We verify this implication in the simulation below.

Conformist pressure reduces the rate of convergence We show through a simulated experiment that conformist preference acts as an inertial term that slows down convergence, and in some extreme cases, is even able to change the survival philosophy.

We consider a log-linear specification for the fundamental value process in simulation, which is slightly different from the linear specification in Assumption 4.[33] The fundamental values of the two styles are given by:

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[32] Psychological factors in which investors' choices depend principally on the behavior of others have been considered in the literature (Lux, 1995; Pedersen, 2022). The key mechanism is similar to Kirman's (1991, 1993) formalization of recruitment in ant populations and Topol's (1991) theory of mimetic contagion.

[33] A linear specification allows us to derive simple closed-form results that highlight the central economic implications of our theory. However, a log-linear specification is convenient in practice because it models X_a and X_b as lognormal distributions, and therefore guarantees that the prices (cumulative returns) do not go negative. The same strategy is also used by Hong et al. (2007).

$$X_a = \exp\left(\mu_a + \beta_a r + \epsilon_a - 1\right),$$
$$X_b = \exp\left(\mu_b + \beta_b r + \epsilon_b - 1\right), \tag{25}$$

where

$$\mu_a = \mu_b = 1, \quad \beta_a = 2, \quad \beta_b = 0.1,$$
$$r \sim N(0, 0.1^2), \quad \epsilon_a \sim N(0, 0.3^2), \quad \epsilon_b \sim N(0, 0.1^2), \tag{26}$$

and N denotes the normal distribution. We set $k = 1$, $W_S = 2$, and $W_F = 1$ without loss of generality. We simulate the evolution of 11 philosophies in $\{0, 0.1, \cdots, 1\}$. Without any conformity pressure, the equilibrium philosophy is $f^e = 0.5$ for endogenous returns.

Fig. 2 shows the evolution of all philosophies over 20,000 generations. The initial population is composed of 90% $f = 0$, and 1% of each $f \in \{0.1, 0.2, \cdots, 1\}$. Figs. 2a–2b represent the case of no conformity pressure, showing that $f = 0.5$ quickly dominates the population. The price-to-fundamental ratio stays fairly close to one after an initial period of fluctuations.

In comparison, Figures (2c)–2f use different levels of conformity pressure. In the process of convergence to $f = 0.5$, other philosophies are popular for extended periods of time. This process may appear as cycles of different popular investment philosophies. Within each period, a certain philosophy is so prevalent in the population that the price-to-fundamental ratios are materially affected, resulting in overpricing for style a and underpricing for style b. In fact, the popular philosophy in one period could potentially create a long streak of high returns as more investors adopt it, but as the popular philosophy changes, investors holding the previously popular philosophy will quickly be wiped out.

In this example, the initial average philosophy in the population is close to 0, and therefore, philosophies with low f will grow more quickly due to the conformity effect. Over time, as the average philosophy λ_t grows larger, other philosophies start to grow in response. The conformity pressure enhances the survival of the popular philosophy at the time, and inhibits the growth of other philosophies.

In our simulation, the ultimately dominant philosophy has the chance to grow because it begins with a large enough population such that it is never wiped out completely. In reality, philosophies like $f = 0.5$ might be eliminated quickly due to conformity pressure. From the evolutionary perspective, mutation would act as insurance for all philosophies to have a chance to grow (see Appendix D.2).

The degree of conformity pressure is likely to be difficult and noisy to measure, but in principle, it can be inferred from textual analysis of social media, or proxies such as the level of adoption of financial innovation (a low amount of innovation might suggest a high degree of conformist preference). Empirical tests for conformist preference could be performed by examining groups with different degrees of conformity pressure, and correlating them with the degree of market efficiency or the speed of convergence after large market shocks.

7.2. Attention to novelty

Opposite in effect to conformist preference is attention to novelty. In attention to novelty, investors are more likely to pay attention to an investment philosophy if it is substantially different from the most popular ones. We modify the population dynamics between two generations in Equation (24) in the following way:

$$X^f_{i,t} = \left[I^f_{i,t} X_{at} + (1 - I^f_{i,t}) X_{bt}\right] \cdot \exp\left[\rho(1 - q^f_{t-1})\right], \tag{27}$$

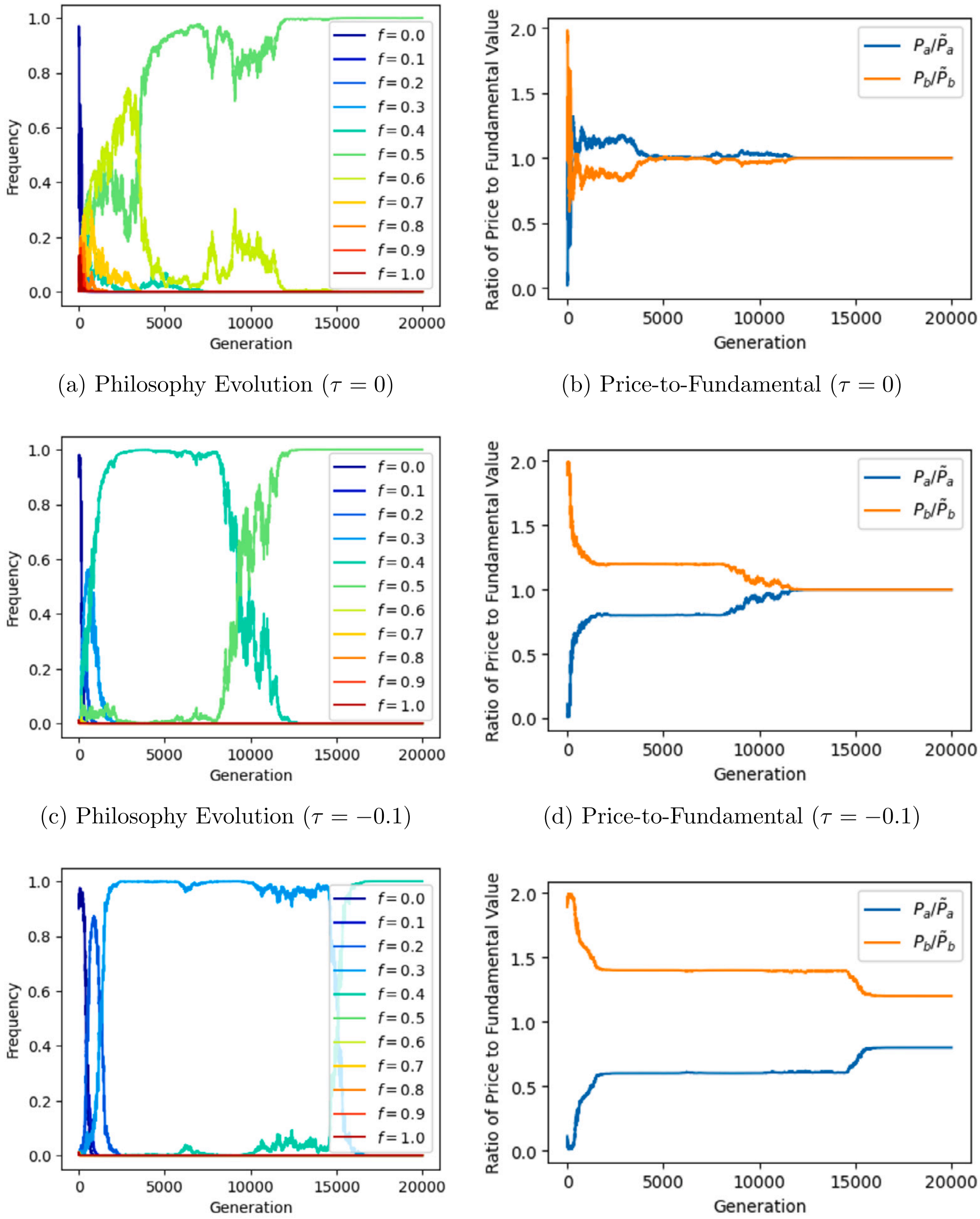
where q^f_{t-1} is the population frequency of type-f investors in generation $t-1$, defined in (15). Here, $\rho >= 0$ represents the degree of attention to novelty. A higher q^f_{t-1} leads to a greater fitness boost due to the attention to novelty.

Attention to novelty adds diversity and leads to "bubbles" We next show that attention to novelty can both add diversity and induce bubbles in market evolution. The existence of bubbles, the mechanism through which they form, and the predictability of their formation and collapse have been an active area of research in recent years (Shiller, 2000; Fama, 2014; Greenwood et al., 2019). Our simulation below provides a potential mechanism for the formation of bubbles within our model.

We use the same simulation specifications as in Equation (25) with 11 philosophies in $\{0, 0.1, \cdots, 1\}$. Fig. 3 shows the simulation paths for different degrees of attention to novelty. Figures (3a)–(3b) show the case with no attention to novelty, and $f = 0.5$ eventually dominates the population. As the degree of attention to novelty increases to 0.1 in Figure (3c), $f = 0.5$ no longer dominates the population. In the long run, there does not exist a single dominant philosophy, because other philosophies are novel compared to the most popular current philosophy and receive a disproportionate conversion in evolution.

In addition, Fig. 3d shows the price-to-fundamental ratio when attention to novelty is set to 0.1. The two investment styles experience repeated episodes of overpricing and underpricing. These patterns of investor composition and asset price dynamics are similar to the bubbles and crashes generated from models of herding (e.g. Lux (1995); Chinco (2023)), as well as return cycles and volatilities generated from learning in markets with multivariate models (e.g. Hong et al. (2007)). Our results provide an alternative channel—attention to novelty—through which such phenomenon can occur.

Finally, we consider a variation of the mechanism specified in Equation (27), by allowing the definition of novelty to include memory. In particular, we replace the term q^f_{t-1} in Equation (27) by:



(a) Philosophy Evolution ($\tau = 0$) (b) Price-to-Fundamental ($\tau = 0$)

(c) Philosophy Evolution ($\tau = -0.1$) (d) Price-to-Fundamental ($\tau = -0.1$)

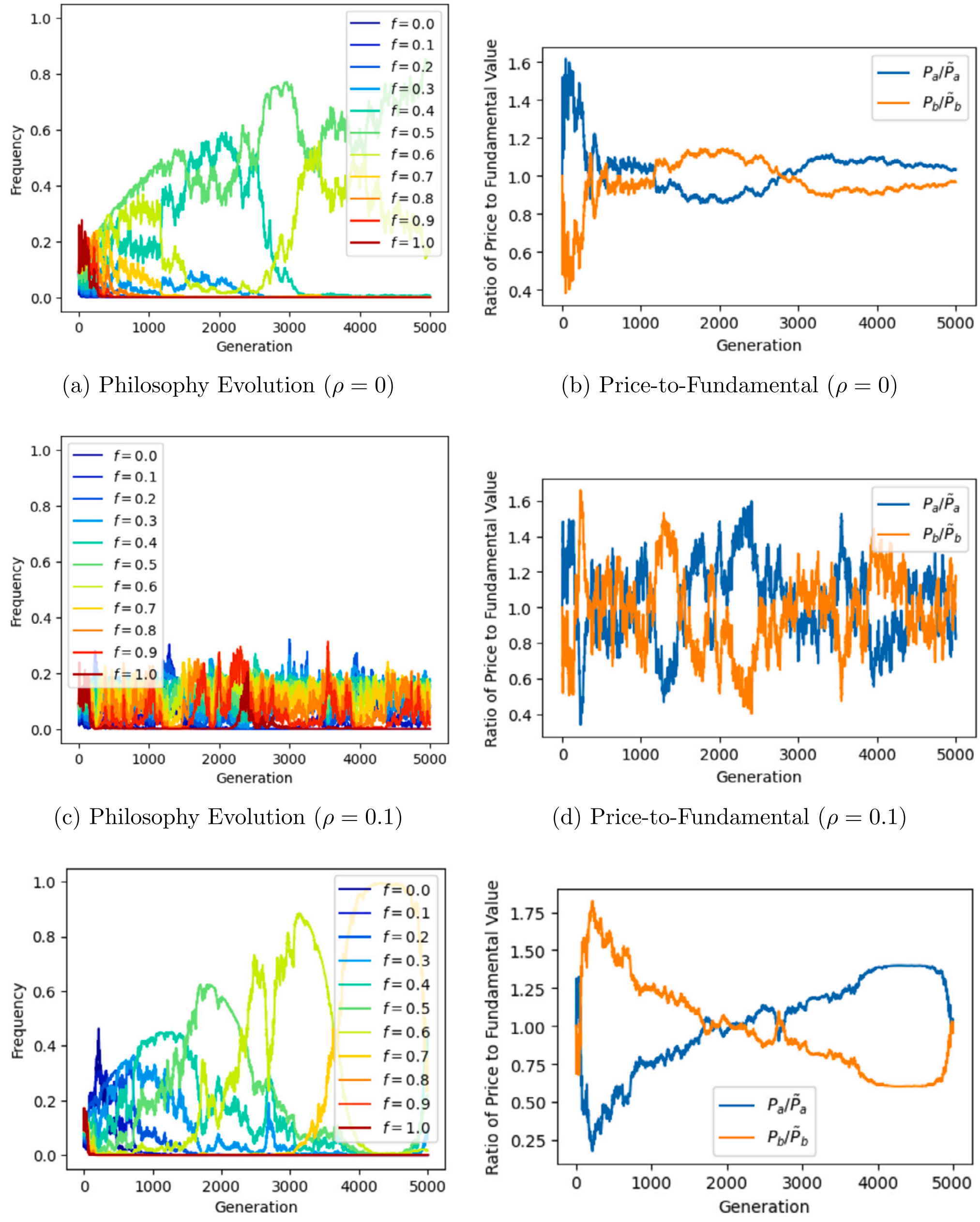
(e) Philosophy Evolution ($\tau = -0.2$) (f) Price-to-Fundamental ($\tau = -0.2$)

Fig. 2. Conformist pressure slows down the rate of convergence. Evolution of philosophies $f \in \{0, 0.1, \cdots, 1\}$ and its corresponding price-to-fundamental ratio over 5000 generations with the environment (style payoffs) specified in (25)–(26). (2a)–(2b) represent no conformity pressure. (2c)–(2f) represent conformity pressures with $\tau = -0.1$ and $\tau = -0.2$.

$$\bar{q}^f_{t-1} = \bar{q}^f_{t-2} \times \text{decay} + q^f_{t-1} \times (1 - \text{decay}).$$

This modified specification captures the fact that investors may view a particular philosophy as novel not just because it has not appeared in the last period, but because it has not appeared for a long time. Here decay is a parameter controlling the length of the memory, which we set to 0.9999 in our simulation.

Figs. 3e–3f demonstrate the evolution of philosophies as well as the price-to-fundamental ratios. With memory, it is even more clear that the population experienced multiple cycles in which popular philosophies alternate. In terms of the equilibrium prices, style b experienced a sharp increase in price in the beginning, leading to a bubble, which slowly bursts over the course of the evolution. Towards the end of our simulation, style b is beginning to inflate another potential bubble. This example further demonstrates that the speed of bubble formation and bursts can be affected by the length of investors' memory.



(a) Philosophy Evolution ($\rho = 0$) (b) Price-to-Fundamental ($\rho = 0$)

(c) Philosophy Evolution ($\rho = 0.1$) (d) Price-to-Fundamental ($\rho = 0.1$)

(e) Philosophy Evolution ($\rho = 0.1$, decay=0.9999) (f) Price-to-Fundamental ($\rho = 0.1$, decay=0.9999)

Fig. 3. Evolution of philosophies $f \in \{0, 0.1, \cdots, 1\}$ over 5000 generations with the environment (style payoffs) specified in (25)–(26). (3a)-(3b) represent no attention to novelty. (3c)-(3f) represents different degrees of attention to novelty.

8. Summary of empirical implications

We summarize here the key empirical implications of our model in a series of predictions justified by specific aspects of our model. The survival of an investment style or a fund is jointly determined by several elements, including its expected return, beta, and volatility. In particular, the scaled alpha—defined as the expected gross return of a style divided by its beta—plays a critical role.

Expected return-related implications

Prediction 1. A fund with higher expected return tends to attract more investors after controlling for other factors such as beta and volatility.

See Propositions 2 and 5.

Beta-related implications

Prediction 2. A fund with lower beta tends to attract more investors when its scaled alpha is comparable with alternative funds, and a fund with higher beta tends to attract more investors when its scaled alpha is much higher than alternative funds, both after controlling for other factors such as expected return and volatility.

See Propositions 3 and 6.

Prediction 3. The "beta puzzle"[34] (i.e., that stocks with high beta earn low expected return) tends to occur when market volatility is low.

According to Propositions 4 and 7, stocks with high beta and low expected return have low scaled alphas, which gains popularity when the common variance, $Var(r)$, decreases. This drives down the returns for stocks with high beta relative to stocks with low beta.

Variance-related implications

Prediction 4. A fund with higher idiosyncratic volatility tends to lose investors, and a fund with lower idiosyncratic volatility tends to attract investors, both after controlling for other factors such as expected return, beta, and market volatility.

See Propositions 4 and 7.

Prediction 5. In volatile markets, investors tend to allocate to stocks and funds with higher scaled alphas. A high scaled alpha can therefore be understood as a defensive characteristic of a fund.

See Propositions 4 and 7.

Prediction 6. The "idiosyncratic volatility puzzle"[35] (i.e., that stocks with high idiosyncratic volatility earn low expected return) tends to occur for stocks with high scaled alpha when market volatility is high, and for stocks with low scaled alpha when market volatility is low.

Because the survival of stocks with high idiosyncratic volatility and low expected return is determined by their betas and the market volatility jointly (see Lemmas 1 and 2), an increase in market volatility for stocks with high scaled alpha makes their survival more likely (see Propositions 4 and 7). The same is true when a decrease in market volatility occurs for stocks with low scaled alpha.

Psychological effects-related implications

Prediction 7. When the degree of conformity pressure in the population is high, asset prices are more likely to deviate from their fundamental values, market efficiency tends to be lower, and the speed of convergence after large market shocks tends to be slower.

See Section 7.1.

Prediction 8. Asset bubbles and bursts are more likely to occur when the degree of attention to novelty in the population is high.

See Section 7.2.
Appendix E discusses potential ways to perform empirical tests on these predictions.

[34] See Baker et al. (2011) and Frazzini and Pedersen (2014).
[35] See Ang et al. (2006, 2009).

9. Conclusion

In a cultural evolutionary model with competing investment philosophies that place different probability weights on two investment styles, we have shown that in equilibrium, the market consists of a mixed population that invests in both investment styles. This implies a wider variation of coexisting strategies than in traditional models, as exemplified by the mutual fund separation theorems deriving from versions of the Capital Asset Pricing Model (Sharpe, 1964; Merton, 1972).

The survival of investment philosophies is jointly determined by several elements, including the asset's mean return, beta, idiosyncratic volatility, and market volatility. We also derive the evolutionary equilibrium investment philosophy with respect to these return characteristics. In general, higher mean returns promote the survival of the investment style, while higher idiosyncratic volatility opposes the survival of the style, and higher common factor volatility promotes the survival of the style with higher scaled alpha, defined as the ratio of the style's alpha to its market beta. These results are similar for both exogenous and endogenous returns.

We extend our model to allow for general replication rules between two consecutive periods, and to incorporate the impact of supply and demand on asset prices in a market equilibrium model. We find that the key implications in terms of the survival of investment philosophies with respect to return characteristics remain robust under these extensions.

We also extend our evolutionary model to include two types of psychological effects that affect investor receptiveness toward the investment philosophies of others. This reinforces our prediction that many competing investment styles and philosophies are able to coexist.

Our results provide one explanation for the long-run evolutionary survival of a wide range of investment styles. For example, there is a variety of investment styles employed in the hedge fund industry with heterogeneous return characteristics (Chan et al., 2006). Our model predicts that investments with high scaled alpha tend to flourish during periods of high volatility. This implies that the popularity and attrition rates of different investment styles will vary in different market environments, and specifically, that high market volatility will promote styles with high scaled alpha. These intuitive implications for the hedge fund industry have been documented empirically by Getmansky et al. (2015).

Our model also offers some possible explanations for certain puzzles about returns that are difficult to reconcile within traditional asset pricing models, leading to several directions for future empirical testing. Our model can partially explain the "beta puzzle" that high beta stocks underperform and low beta stocks outperform (Baker et al., 2011; Frazzini and Pedersen, 2014), because strategies that invest in stocks with high beta and low expected return can survive in the long run, especially when the market volatility is low. Our model also offers a partial explanation for the "idiosyncratic volatility puzzle," that stocks with high idiosyncratic risk earn low returns (Ang et al., 2006, 2009), because investment styles that allocate to these stocks can survive in the long run, provided they have low betas—and therefore high scaled alpha—when market volatility is high.

Our model can be extended to further explore social contagion and its implications for investment styles and investor behaviors, for instance, to include resource constraints, which may generate strategic interactions, autocorrelated environments, which may generate intelligent behaviors with memory, and overlapping investors operating at different frequencies (resembling high-frequency and long-term investors), which may further generate price momentum and bubbles. More generally, the evolutionary finance approach behind our model offers a possible framework for modeling how social contagion causes these behaviors and market phenomena.

Appendix A. Comparative statics: an example

We apply the results in Section 4 to study a few common competing investment styles. In particular, we consider a special case in which returns are further specified by:

Assumption 7. Investment styles a and b have the same mean return, and style a has a higher beta and a higher style-specific variance than style b:

$$\mu_a = \mu_b; \quad \beta_a > \beta_b; \quad Var(\epsilon_a) > Var(\epsilon_b).$$

Style a has higher systematic risk and higher volatility than style b. This specification is suggestive of several possible real-world applications, such as active versus passive investing, or investing by high versus low income (high versus low dividend yield). Another application is the so-called defensive investing aimed at stocks with low volatility, a common smart beta strategy. For example, AQR offers funds marketed as "defensive" that are designed to focus on low volatility stocks. We will call a the "riskier" style and b the "safer" style.

It immediately follows that the scaled alpha is higher for style b:

$$\frac{\mu_a}{\beta_a} < \frac{\mu_b}{\beta_b},$$

and Lemma 1 reduces to:

$$\mathbb{E}[y] = \mathbb{E}\left[\frac{X_a}{X_b}\right] \approx 1 + \left(\frac{\beta_a \beta_b^2}{\mu_b^3}\right)\left(\frac{\mu_a}{\beta_a} - \frac{\mu_b}{\beta_b}\right)\mathrm{Var}(r) + \left(\frac{\mu_a}{\mu_b^3}\right)\mathrm{Var}(\epsilon_b),$$

$$\mathbb{E}[1/y] = \mathbb{E}\left[\frac{X_b}{X_a}\right] \approx 1 + \left(\frac{\beta_a^2 \beta_b}{\mu_a^3}\right)\left(\frac{\mu_b}{\beta_b} - \frac{\mu_a}{\beta_a}\right)\mathrm{Var}(r) + \left(\frac{\mu_b}{\mu_a^3}\right)\mathrm{Var}(\epsilon_a) > 1.$$

Up to a second-order Taylor approximation, $\mathbb{E}[1/y]$ is always greater than 1, which implies that style a alone is never an equilibrium. The long-run equilibrium philosophy is either purely style b (with a higher scaled alpha), or a combination of both investment styles, in which case the first-order condition for f in Lemma 2 reduces to:

$$0 = \left[f\beta_a + (1-f)\beta_b\right]\left(\beta_b - \beta_a\right)Var(r) + (1-f)Var(\epsilon_b) - fVar(\epsilon_a),$$

from which the evolutionary equilibrium philosophy f^* can be solved. We summarize these observations as follows:

Proposition A.1. *Under Assumptions 1–4 and 7, up to a second-order Taylor approximation with respect to r, ϵ_a and ϵ_b, style a alone is never an evolutionary equilibrium. Style b alone is evolutionary equilibrium if*

$$Var(\epsilon_b) < \beta_b(\beta_a - \beta_b)Var(r). \tag{A.1}$$

Otherwise the population consists of investors in both styles in the long run, and the equilibrium fraction of investors in style a is given by:

$$f^* = \frac{Var(\epsilon_b) - \beta_b(\beta_a - \beta_b)Var(r)}{Var(\epsilon_a) + Var(\epsilon_b) + (\beta_a - \beta_b)^2 Var(r)}. \tag{A.2}$$

It is evident from Proposition A.1 that the population tends to have only investors in style b when the common component has a high volatility ($Var(r)$), the safer style has a low volatility ($Var(\epsilon_b)$), and the riskier style has a high beta (β_a). In the case that the population consists of investors in both styles, the fraction of investors in style a increases as the variance of the a-specific component ($Var(\epsilon_a)$) decreases, the variance of the b-specific component ($Var(\epsilon_b)$) increases, and the variance of the common component ($Var(r)$) decreases. This is consistent with our earlier discussions indicating that risk tends to reduce the evolutionary success of a style.

When comparing the riskier style and the safer style, Proposition A.1 implies that the riskier style alone is never optimal. A certain amount of allocation in the safer style is always desirable. It also implies that allocation in the riskier style tends to increase in stable environments and decrease in volatile markets.

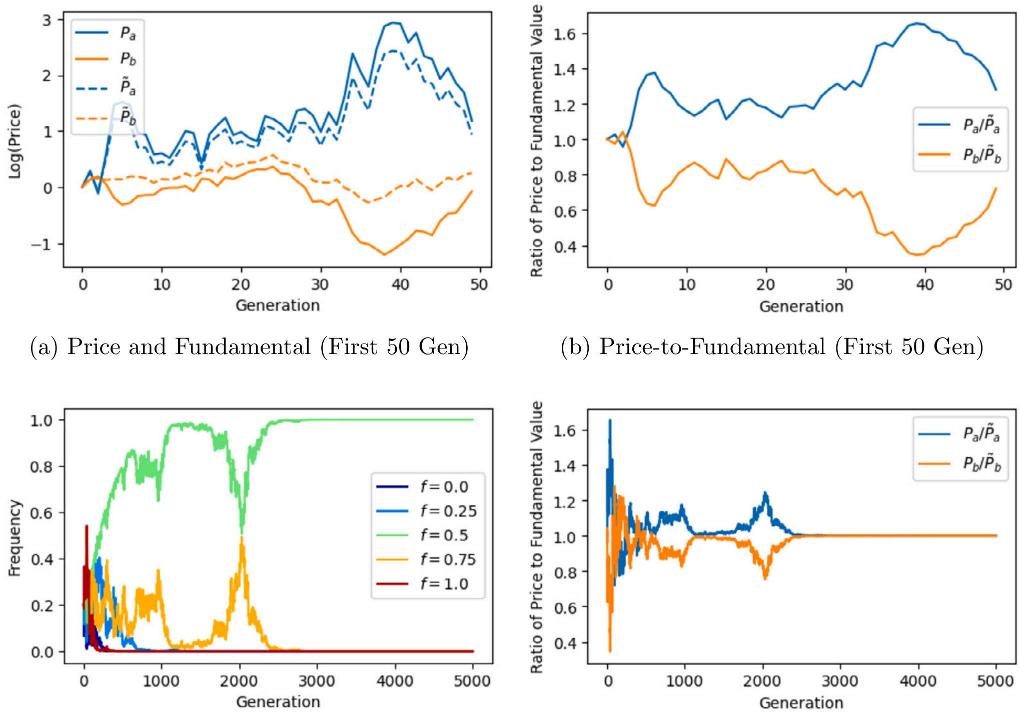
Appendix B. Additional results for market equilibrium

Table A.1 shows the equilibrium prices when different philosophies survive in equilibrium for several different values of k, the elasticity of supply with respect to price deviations from the fundamental value.

We then provide additional simulation examples to demonstrate the effect in market equilibrium. We consider a market in which investment returns are given by the same specification in Equation (25) as the simulated example in the main paper. Fig. A.1 demonstrates a market in which prices are determined endogenously, with five philosophies ($f = 0, 0.25, 0.5, 0.75, 1$) over 5,000 generations. Figs. A.1a–A.1b focuses on the first 50 generations, and show the (log)-endogenous price, the (log)-fundamental value, and the price-to-fundamental ratio, respectively. Prices fluctuate around the fundamental value. Style a is overpriced in this period due to its high demand initially. Fig. A.1c shows the evolution of five philosophies ($f = 0, 0.25, 0.5, 0.75, 1$) over 5,000 generations, in which the vertical axis denotes the frequency of each type of investor in the population. $f = 1.0$ is popular for a short period of time in the very beginning, consistent with the fact that style a is over-priced in Figs. A.1a–A.1b. After that, the equilibrium philosophy $f^* = 0.5$ quickly dominates the population. Finally, Fig. A.1d shows the price-to-fundamental ratio over the entire course

Table A.1
A comparison between the ratio of surviving philosophies and the ratio of equilibrium prices for different levels of the elasticity of supply with respect to price deviations from the fundamental value.

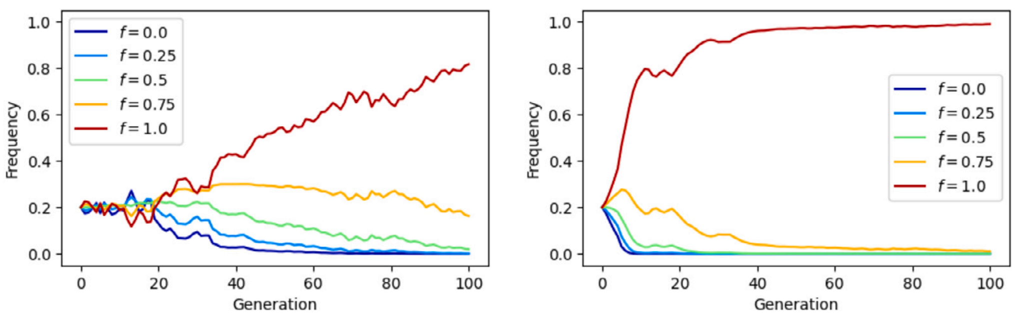
Elasticity of supply k	Ratio of philosophies f_1^e/f_2^e	Ratio of prices $P_{a,t,1}/P_{a,t,2}$
	5	1.17
	2	1.07
10	1	1.00
	0.5	0.93
	0.2	0.85
	5	1.38
	2	1.15
5	1	1.00
	0.5	0.87
	0.2	0.72
	5	2.24
	2	1.41
2	1	1.00
	0.5	0.71
	0.2	0.45



(a) Price and Fundamental (First 50 Gen) (b) Price-to-Fundamental (First 50 Gen)

(c) Philosophy Evolution (d) Price-to-Fundamental

Fig. A.1. A demonstration of market equilibrium in which prices and returns are determined endogenously. (A.1a) and (A.1b) show the fundamental value, price, and price-to-fundamental ratio over the first 50 generations in evolution. (A.1c) and (A.1d) show the equilibrium philosophy f^e and the price-to-fundamental ratio over 5,000 generations.



(a) Philosophy Evolution (Exogenous Returns) (b) Philosophy Evolution (Endogenous Returns)

Fig. A.2. Market equilibrium speeds up the rate of convergence. The evolution of the equilibrium philosophy f^* with exogenous returns (A.2a) and the equilibrium philosophy f^e with endogenous returns (A.2b) are shown over 100 generations.

of the evolution. After an extended period of fluctuations, the ratio eventually converges to one. In reality, the market conditions are constantly changing. Instead of the long-run limit, the short-term oscillation shown here may be typical of the market.

In a slightly different simulation experiment, we increase the mean return of style a so that in equilibrium $f^e = 1$ is the dominant behavior:

$$\begin{aligned} &\mu_a = 1.1, \quad \mu_b = 1, \quad \beta_a = 2, \quad \beta_b = 0.1, \\ &r \sim N(0, 0.1^2), \quad \epsilon_a \sim N(0, 0.3^2), \quad \epsilon_b \sim N(0, 0.1^2), \end{aligned} \tag{A.3}$$

where N denotes the normal distribution. We also set $k = 0.3$, $W_S = 1.2$, and $W_F = 1$.

Fig. A.2 shows that market equilibrium prices may speed up the rate of convergence, by comparing the evolution of the same five philosophies ($f = 0, 0.25, 0.5, 0.75, 1$) when returns are exogenously determined by the fundamental value (Fig. A.2a), and when returns are endogenously determined by market equilibrium (Fig. A.2b). In the former, the market still contains multiple philosophies after 100 generations, while in the latter, $f^e = 1.0$ dominates the population after around 50 generations.

This phenomenon can be understood by the expression of equilibrium returns in Equation (21). When the aggregate demand is, for example, increasing for style a, market equilibrium forces further enhance the returns for that style. In this sense, market equilibrium serves as a sort of momentum for style returns, thereby helping the dominant style to dominate faster. The same mechanism is also adopted in the computer science literature for optimizing the loss function of deep neural networks.[36]

Appendix C. Generalization for multiple styles

Our main model in Section 3 considers two competing investment styles whose returns share a common factor. The simplicity of this specification allows us to derive closed-form expressions that highlight many key economic insights. However, our model can be substantially generalized to include multiple investment styles. We describe this extension here.

Consider investors who choose from m investment styles (or assets), $\{1, \cdots, m\}$, and this results in one of m corresponding random payoffs, $(X_1, \cdots, X_m)$. Suppose each individual chooses style i with probability p_i, for $i = 1, 2, \cdots, m$. Let $\mathbf{p} = (p_1, \cdots, p_m)$ be the probability vector that characterizes an individual's investment philosophy. $\mathbf{p}$ satisfies the following conditions:

$$0 \le p_i \le 1, \quad \forall i = 1, \cdots, m$$

$$\sum_{i=1}^{m} p_i = 1.$$

The style returns are determined by the following factor structure:

$$\begin{cases} X_1 = \mu_1 + \beta_1 r + \epsilon_1 \\ \cdots \\ X_m = \mu_m + \beta_m r + \epsilon_m. \end{cases}$$

For simplicity, we write $\mathbf{X} = (X_1, \dots, X_m)$ to denote the vector of all style returns. In the multinomial choice model, the population growth rate is determined by the vector $\mathbf{p}$. Therefore, it is convenient to consider the number of offspring for individual i with type $f = \mathbf{p}$:

$$X_i^{\mathbf{p}} = I_{1,i}^{\mathbf{p}} X_{1,i} + \cdots + I_{m,i}^{\mathbf{p}} X_{m,i}$$

where $(I_1^{\mathbf{p}}, \cdots, I_m^{\mathbf{p}})$ is the multinomial indicator variable with probability $\mathbf{p} = (p_1, \cdots, p_m)$:

$$(I_1^{\mathbf{p}}, \cdots, I_m^{\mathbf{p}}) = \begin{cases} (1, 0, \cdots, 0) & \text{with probability } p_1 \\ (0, 1, \cdots, 0) & \text{with probability } p_2 \\ \cdots \\ (0, 0, \cdots, 1) & \text{with probability } p_m. \end{cases}$$

We denote the total number of type $\mathbf{p}$ individuals in generation T by $n_T^{\mathbf{p}}$. Similar to our main model, we can characterize the log-geometric-average growth rate of type $\mathbf{p}$ in the general m-choice setting. As the number of generations and the number of individuals in each generation increases without bound, $T^{-1} \log n_T^{\mathbf{p}}$ converges in probability to the log-geometric-average growth rate

$$\mu(\mathbf{p}) = \mathbb{E}\left[\log\left(\mathbf{p}\mathbf{X}'\right)\right]. \tag{A.4}$$

The next result gives a necessary and sufficient condition for investment philosophies to be optimal.

Proposition A.2. $(p_1^*, \cdots, p_m^*)$ *maximizes (A.4) if and only if*

$$\mathbb{E}\left[\frac{p_1 X_1 + \cdots + p_m X_m}{p_1^* X_1 + \cdots + p_m^* X_m}\right] \le 1, \quad \forall (p_1, \cdots, p_m). \tag{A.5}$$

The next result generalizes Proposition 1, and characterizes the optimal type $\mathbf{p}^*$ that maximizes Equation (A.4).

Proposition A.3. *Under assumptions (A1)-(A3) generalized to multiple styles, the optimal factor loading $\mathbf{p}^* = (p_1^*, \cdots, p_m^*)$ that maximizes (A.4) is given by:*

[36] See, for example, the adaptive momentum (Adam) algorithm (Kingma and Ba, 2015).

$$\mathbf{p}^* = \begin{cases} (1,0,\cdots,0) & \text{if } \mathbb{E}\left[\frac{X_2}{X_1}\right] < 1, \mathbb{E}\left[\frac{X_3}{X_1}\right] < 1, \cdots, \mathbb{E}\left[\frac{X_m}{X_1}\right] < 1 \\ (0,1,\cdots,0) & \text{if } \mathbb{E}\left[\frac{X_1}{X_2}\right] < 1, \mathbb{E}\left[\frac{X_3}{X_2}\right] < 1, \cdots, \mathbb{E}\left[\frac{X_m}{X_2}\right] < 1 \\ \cdots & \\ (0,0,\cdots,1) & \text{if } \mathbb{E}\left[\frac{X_1}{X_m}\right] < 1, \mathbb{E}\left[\frac{X_2}{X_m}\right] < 1, \cdots, \mathbb{E}\left[\frac{X_{m-1}}{X_m}\right] < 1 \\ \text{solution to (A.7)} & \text{otherwise.} \end{cases} \tag{A.6}$$

In the last case, suppose without loss of generality that $\mathbf{p}^* = (p_1^*, \cdots, p_l^*, 0, \cdots, 0)$. In other words, only the first l alphas are zero. Then $\mathbf{p}^*$ in the last case of (A.6) is defined implicitly by:

$$\mathbb{E}\left[\frac{X_1}{p_1^* X_1 + \cdots + p_l^* X_l}\right] = \cdots = \mathbb{E}\left[\frac{X_l}{p_1^* X_1 + \cdots + p_l^* X_l}\right] = 1, \tag{A.7}$$

and $\mathbf{p}^*$ satisfies:

$$\begin{cases} \mathbb{E}\left[\frac{X_{l+1}}{p_1^* X_1 + \cdots + p_l^* X_l}\right] < 1 \\ \cdots \\ \mathbb{E}\left[\frac{X_m}{p_1^* X_1 + \cdots + p_l^* X_l}\right] < 1. \end{cases} \tag{A.8}$$

Proposition A.3 asserts that $\mathbf{p}^* = (p_1^*, \cdots, p_l^*, 0, \cdots, 0)$ is optimal if and only if the expectation of any irrelevant style divided by the optimal combination of styles is less than 1, and any style in the optimal combination divided by the optimal combination is equal to 1.

Proposition A.3 generalizes Proposition 1 in our main model. Comparative statics results with respect to mean return, beta, and volatilities can therefore be carried out in principle. In particular, in the first m cases of Equation (A.6) when there is a single dominant style, the conditions are very similar to those in Proposition 1.[37] Therefore, we have the following comparative statics for style 1-investors, without loss of generality, which generalizes our results in Section 4.

Proposition A.4 *(Comparative Statics for Multiple Styles). Under Assumptions 1–4 generalized to multiple styles, style 1-investors tend to dominate the population if:*

(i) *the mean return of style 1, μ_1, increases;*
(ii) *the mean return of style k, μ_k, decreases, for $k = 2, 3, \ldots, m$;*
(iii) *the sensitivity of style k to the common component, β_k, increases, for $k = 2, 3, \ldots, m$;*
(iv) *the sensitivity of style 1 to the common component, β_1, increases, conditional on style 1's scaled alpha being sufficiently greater than all other styles' scaled alphas:*

$$\frac{\mu_1/\beta_1}{\max_{k\neq 1}\{\mu_k/\beta_k\}} > 2;$$

(v) *the sensitivity of style 1 to the common component, β_1, decreases, conditional on style 1's scaled alpha being sufficiently small relative to all other styles' scaled alphas:*

$$\frac{\mu_1/\beta_1}{\min_{k\neq 1}\{\mu_k/\beta_k\}} < 2;$$

(vi) *the variance of style-specific component for style 1, $Var(\epsilon_1)$, decreases;*
(vii) *the variance of the common component, $Var(r)$, increases, conditional on style 1's scaled alpha being greater than all other styles' scaled alphas:*

$$\frac{\mu_1}{\beta_1} > \max_{k\neq 1} \frac{\mu_k}{\beta_k};$$

(viii) *the variance of the common component, $Var(r)$, decreases, conditional on style 1's scaled alpha being smaller than all other styles' scaled alphas:*

$$\frac{\mu_1}{\beta_1} < \min_{k\neq 1} \frac{\mu_k}{\beta_k}.$$

The conditions in (iii)–(viii) hold up to second-order Taylor approximation with respect to r, ϵ_a and ϵ_b.

[37] However, when the evolutionary equilibrium philosophy involves a mix of multiple investment styles, the condition in Equation (A.7) defines $\mathbf{p}^*$, but the analytic comparative statics become intractable.

Propositions A.3–A.4 together show that our results on co-existence of investment styles and their comparative statics in Section 4 hold true in the multi-style setting.

Appendix D. Additional discussions on psychological bias

D.1. Population growth with psychological bias

We first consider population dynamics with conformist preference as specified in Equation (24). By a similar derivation as in Equation (3), the population size of type-f investors in period T is:

$$\begin{aligned} n_T^f &= \prod_{t=1}^{T} \left[f X_{at} + (1-f) X_{bt} \right] \exp\left[\tau (f - \lambda_{t-1})^2 \right] \\ &= \exp\left\{ \sum_{t=1}^{T} \log\left[f X_{at} + (1-f) X_{bt} \right] + \tau \sum_{t=1}^{T} (f - \lambda_{t-1})^2 \right\}. \end{aligned}$$

Taking the logarithm of the number of offspring, we have:

$$\lim_{T\to\infty} \frac{1}{T} \log n_T^f = \mathbb{E}[\log\left(f X_a + (1-f) X_b \right)] + \tau \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} (f - \lambda_{t-1})^2, \tag{A.9}$$

where the first term is simply the log-geometric average growth rate of the population without conformity pressure, $\alpha(f)$, in Equations (4). From Equations (24) and (A.9), we can see that the magnitude of the conformity pressure τ acts roughly as a multiplicative factor in the fitness, or an additive factor in the population growth rate.[38]

The case of attention to novelty as specified in Equation (27) is similar to the case of conformity. The logarithm of the population size is:

$$\lim_{T\to\infty} \frac{1}{T} \log n_T^f = \mathbb{E}[\log\left(f X_a + (1-f) X_b \right)] + \rho \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} (1 - q_{t-1}^f), \tag{A.10}$$

where the first term is again the log-geometric average growth rate of the population without attention to novelty, $\alpha(f)$, in Equation (4).[39] Suppose a long time has passed, and that a philosophy almost dominates the population. The second term in Equation (A.10) is close to 0 for that philosophy, while other philosophies receive a fitness boost due to the attention to novelty, and may tend to outgrow the currently popular philosophy. Therefore, it is hard for any single philosophy to dominate in the long run.

D.2. Diverse investment philosophies via mutation

In our main model, we have derived the evolutionary equilibrium investment philosophy and demonstrated the survival of diverse investment *styles* in the long run, using both endogenous and exogenous style returns. Diverse investment philosophies can coexist in the long run with psychological effects such as attention to novelty. In the model, the investment philosophy f is assumed to be perfectly heritable across agents. We note here that by introducing mutation in investment philosophy f between two periods, as modeled by Brennan et al. (2018) in a different context, our framework can achieve diversity in investment philosophies in equilibrium with only return-based replication rules.

Positive mutation rates lead to the survival of a mix of investment philosophies f, which in unstable financial environments is important to rescue unpopular philosophies from extinction. In fact, depending on the degree of environmental instability, there is an evolutionary equilibrium mutation rate found by maximizing the population growth as a whole in the long run, as shown in the model of Brennan et al. (2018). Thus, in highly unstable financial environments, the mutation rate should be higher, and a high degree of diversity in investment philosophies will be evolutionarily desirable for higher growth rates of the total population. In relatively stable financial environments, the mutation rate will be lower, which implies a low degree of diversity in investment philosophies. The diversity in investment philosophies is determined by market selection to match the degree of environmental instability.

There are several possible ways to estimate environmental instability empirically. For example, one can track the time variation in volatility using the VIX index as a proxy, or different interest rate environments using the US federal funds rate as a proxy. Future research should find it of interest to test whether a higher frequency of environmental change is associated with a higher degree of diversity in investment philosophies.

D.3. Tradeoffs in social learning

Conformist preference and attention to novelty have opposite effects in social learning: one promotes learning from other people and bets on the “wisdom of crowds”, while the other encourages novel and contrarian ideas.

[38] However, we cannot apply the Law of Large Numbers to the second term of (A.9) in general to determine an explicit solution in the limit because λ_{t-1} is nonstationary.

[39] Likewise, we cannot apply the Law of Large Numbers to the second term in general to obtain an explicit solution in the limit because q_{t-1}^f is nonstationary.

When the degree of conformist preference is extreme, we have seen that the convergence to the long-run equilibrium investment philosophy can be greatly delayed (see Fig. 2). This is not surprising, as the "wisdom of crowds" only works under the assumption that individuals have different information sources and relatively independent decision-making processes. If this condition is violated, the "effective population size" (to borrow a term from population genetics) is greatly reduced, and crowds may have little wisdom.

On the other extreme, when the degree of attention to novelty is high, investment philosophies that work well in the current environment have a weaker influence on the adoption of philosophies in the future. Investors no longer use the information from past returns embedded in the population frequencies. As a result, no one benefits from the "wisdom of crowds", which can lead to bubbles and bursts (see Fig. 3).

In practice, an intermediate amount of social learning is probably most desirable from the perspective of adopting the fittest philosophy in the current environment. For example, studies on interactions between financial traders have documented a large range of rates of idea flow, from isolated individual traders at one end to traders trapped in an echo chamber at the other end, finding that the best investment performance is achieved between the two extremes (Altshuler et al., 2012; Pan et al., 2012).

Appendix E. Strategies for empirical testing

Empirical testing requires estimating the investment philosophy f^* and the characteristics of its style returns. In this section, we will discuss several possible ways to perform empirical tests on these predictions, including estimation methods and the use of large-scale datasets.

Style returns The evolutionary model can be applied to various types of investment styles, taken in pairs a and b, value versus growth styles being an example. Following the notation in Assumption 4, it is straightforward to use market data to estimate the expected returns, μ_a and μ_b, and the market loadings, β_a and β_b, by regressing the time series of observed style returns on market returns. In addition, one can estimate the variance of the common return component through the sample variance of the market, and estimate the variance of the idiosyncratic return component through the sample variance of the residuals from the regression.

Population style proportions If the financial environment is stable, the investment philosophy f^* corresponds to the proportion of style-a investors in the population. In the example of value versus growth styles, this would correspond to the proportion of value versus growth investors in the population, which can be estimated by textual analysis of investing social media or blogging sites such as SeekingAlpha and StockTwits.[40] For example, Cookson and Niessner (2020) studies disagreement among investors on a social media investing platform, in which users regularly express their opinions about the same stocks, and where user profile information explicitly conveys the user's broad investment approach (such as value versus growth, or fundamental versus technical).

Other possible data sources for estimating the frequencies of investors using different styles include the mutual fund industry and the exchange-traded fund (ETF) market. Barber et al. (2016) and Berk and Van Binsbergen (2016), for example, document that mutual fund flows are related to past performance in terms of excess returns (alpha) and factor loadings. The ETF market has grown at a feverish pace, and there are now thousands of different ETFs, each focusing on a unique investment style (Ferri, 2011; Lettau and Madhavan, 2018). This includes regional and industry-specific ETFs, such as ETFs holding stocks in developed versus developing countries, style-specific ETFs, such as value versus momentum ETFs, and fundamentals-driven versus AI-powered ETFs. The assets under management of these ETFs provide a possible proxy for the aggregate investor frequencies in those investment styles, with inflow and outflow of assets as a proxy for change over time.

Hedge funds Hedge funds are a fast-growing sector of the financial services industry. One of its attractions for investors is generating returns with a relatively low correlation with traditional investment asset classes. Hedge funds are also perceived by many to draw the smartest and most innovative money managers, owing to the investment flexibility and low level of regulation relative to other financial management vehicles. With relatively low barriers to entry and exit, the hedge fund sector is a highly competitive industry. Based on these unique characteristics, hedge funds are particularly suitable for the empirical study of market selection.

Two data sources are available for empirical tests in the hedge fund industry, the Credit Suisse Hedge Fund Index and the Lipper/TASS Hedge Fund database. The first of these tracks approximately 9,000 funds, and reflects the monthly net performance in several fund categories, such as Convertible Arbitrage, Event Driven, Long/Short Equity, Global Macro, and Managed Futures. The Lipper/TASS Hedge Fund database contains performance data on over 18,000 actively reporting and "graveyard" hedge funds, including their investment styles, returns, births and deaths, and assets under management.

To test the implications of Propositions 2–14, style returns can be estimated, either directly from the Credit Suisse Hedge Fund Index, or by sampling individual hedge funds from the Lipper/TASS Hedge Fund database following a particular investment style. The common and idiosyncratic components of the style returns can be decomposed by regressing them against common financial and macroeconomic factors (see Fung and Hsieh (2004), Hasanhodzic and Lo (2006), and Bali et al. (2011) for examples). Furthermore, the proportion of hedge funds engaged in each style can be tracked over time from the Lipper/TASS Hedge Fund database. Together this data would provide the information needed to test the predicted relationships between the proportion of investors who are attracted to each investment style, and return characteristics such as mean, beta, common variance, and idiosyncratic variance.

[40] Trading data alone is not fully informative about population frequencies, owing to market clearing. For example, in the case in which all investors are identical growth investors or identical value investors, there will be no trades, and identical trading outcomes are impossible to distinguish.

Social networks and psychology With the collection of "Big Data" in the digital era, another promising financial data source is social media.[41] Modern digital data includes information about call records, credit card transactions, and social network usage, among other recorded interactions. This data is particularly useful to measure social transmission effects such as conformist preference and attention to novelty in our model.

Empirical tests for the effects of attention to novelty are possible using proxies for attention that have been applied in the empirical finance literature (see, for example, Barber and Odean (2007), Da et al. (2011), and Li and Yu (2012)). Henderson and Pearson (2011) find evidence that firms issue certain retail structured equity products with negative expected returns, potentially shrouding some aspects of securities innovation or introducing complexity to attract attention, therefore exploiting uninformed investors. This suggests that some investors do invest based on attention to novelty even if the financial security might not deliver desirable returns, which is consistent with our assumptions.

Appendix F. Proofs

Proof of Proposition 1. This is first proved by Brennan and Lo (2011) and we reproduce the proof here for completeness. This follows from the first and second derivatives of Equation (4). Because the second derivative is strictly negative, there is exactly one maximum value obtained in the interval $[0,1]$. The values of the first-order derivative of $\alpha(f)$ at the endpoints are given by:

$$\alpha'(0) \;=\; \mathbb{E}[X_a/X_b] - 1 \;,\;\; \alpha'(1) \;=\; 1 - \mathbb{E}[X_b/X_a]\,.$$

If both are positive or both are negative, then $\alpha(f)$ increases or decreases, respectively, throughout the interval and the maximum value is attained at $f=1$ or $f=0$, respectively. Otherwise, $f=f^*$ is the unique point in the interval for which $\alpha'(f)=0$, where f^* is defined in Equation (6), and it is at this point that $\alpha(f)$ attains its maximum value. The expression in Equation (5) summarizes the results of these observations for the various possible values of $\mathbb{E}[X_a/X_b]$ and $\mathbb{E}[X_b/X_a]$. Note that the case $\mathbb{E}[X_a/X_b]\le 1$ and $\mathbb{E}[X_b/X_a]\le 1$ is not considered because this set of inequalities implies that $\alpha'(0)\le 0$ and $\alpha'(1)\ge 0$, which is impossible since $\alpha''(f)$ is strictly negative. □

Proof of Proposition 2. $\mathbb{E}[1/y]$ as given in Equation (8) is a decreasing function of μ_a and an increasing function of μ_b. □

Proof of Lemma 1. According to the discussion leading to Lemma 1, calculations of second-order derivatives of $y(r,\epsilon_a,\epsilon_b)$ suffice. For simplicity, we use $(0,0,0)$ to represent $r=\epsilon_a=\epsilon_b=0$.

$$\begin{aligned}
\frac{\partial y}{\partial r} &= \frac{\beta_a(\mu_b+\beta_b r+\epsilon_b)-\beta_b(\mu_a+\beta_a r+\epsilon_a)}{(\mu_b+\beta_b r+\epsilon_b)^2} = \frac{\beta_a\mu_b-\beta_b\mu_a+\beta_a\epsilon_b-\beta_b\epsilon_a}{(\mu_b+\beta_b r+\epsilon_b)^2}\\
\frac{\partial^2 y}{\partial r^2} &= \frac{-2\beta_b(\beta_a\mu_b-\beta_b\mu_a+\beta_a\epsilon_b-\beta_b\epsilon_a)}{(\mu_b+\beta_b r+\epsilon_b)^3} \overset{(0,0,0)}{=\!=} \frac{2\beta_b(\beta_b\mu_a-\beta_a\mu_b)}{\mu_b^3}\\
\frac{\partial y}{\partial \epsilon_a} &= \frac{1}{\mu_b+\beta_b r+\epsilon_b}, \quad \frac{\partial^2 y}{\partial \epsilon_a^2}=0\\
\frac{\partial y}{\partial \epsilon_b} &= -\frac{\mu_a+\beta_a r+\epsilon_a}{(\mu_b+\beta_b r+\epsilon_b)^2}\\
\frac{\partial^2 y}{\partial \epsilon_b^2} &= \frac{2(\mu_a+\beta_a r+\epsilon_a)}{(\mu_b+\beta_b r+\epsilon_b)^3} \overset{(0,0,0)}{=\!=} \frac{2\mu_a}{\mu_b^3}.
\end{aligned}$$

Therefore,

$$\mathbb{E}[y] \approx \frac{\mu_a}{\mu_b} + \frac{\beta_b(\beta_b\mu_a-\beta_a\mu_b)}{\mu_b^3}\mathbb{E}[r^2] + \frac{\mu_a}{\mu_b^3}\mathbb{E}[\epsilon_b^2] = \frac{\mu_a}{\mu_b} + \frac{\beta_a\beta_b^2}{\mu_b^3}\left(\frac{\mu_a}{\beta_a}-\frac{\mu_b}{\beta_b}\right)\mathrm{Var}(r) + \frac{\mu_a}{\mu_b^3}\mathrm{Var}(\epsilon_b), \tag{A.11}$$

which completes the proof of the first part. The approximation for $\mathbb{E}[1/y]$ follows from similar calculations. □

Proof of Proposition 3. According to Lemma 1, $\mathbb{E}[1/y]$ is a decreasing function of β_b; it is a quadratic function of β_a and therefore turns at its vertex. □

Proof of Proposition 4. It follows directly from Lemma 1. □

Proof of Proposition 5. The first-order condition as given in Equation (9) is a decreasing function of f, an increasing function of μ_a, and a decreasing function of μ_b. Therefore, as μ_a increases, the solution f^* has to increase. Similarly, as μ_b decreases, the solution f^* has to increase. □

[41] Some examples of such social media services include SeekingAlpha, StockTwits (used in Cookson and Niessner (2020) and Argarwal et al. (2018)), eToro (used in Altshuler et al. (2012), Pan et al. (2012), and Pentland (2015)), and an unnamed European social trading platform used in Ammann and Schaub (2021).

Proof of Lemma 2. For notational convenience, we let:

$$F(r,\epsilon_a,\epsilon_b) \equiv \frac{(\mu_a-\mu_b)+(\beta_a-\beta_b)r+(\epsilon_a-\epsilon_b)}{[f\mu_a+(1-f)\mu_b]+[f\beta_a+(1-f)\beta_b]r+[f\epsilon_a+(1-f)\epsilon_b]}.$$

The first-order condition reduces to $\mathbb{E}\left[F(r,\epsilon_a,\epsilon_b)\right]=0$, and it suffices to calculate the second-order derivatives of $F(r,\epsilon_a,\epsilon_b)$:

$$\begin{aligned}
\frac{\partial F}{\partial r} &= \frac{(\beta_a-\beta_b)\left\{[f\mu_a+(1-f)\mu_b]+[f\epsilon_a+(1-f)\epsilon_b]\right\}-[f\beta_a+(1-f)\beta_b][(\mu_a-\mu_b)+(\epsilon_a-\epsilon_b)]}{\left\{[f\mu_a+(1-f)\mu_b]+[f\beta_a+(1-f)\beta_b]r+[f\epsilon_a+(1-f)\epsilon_b]\right\}^2}\\
\frac{\partial^2 F}{\partial r^2} &\overset{(0,0,0)}{=\!=} \frac{-2[f\beta_a+(1-f)\beta_b]\left\{(\beta_a-\beta_b)[f\mu_a+(1-f)\mu_b]-[f\beta_a+(1-f)\beta_b](\mu_a-\mu_b)\right\}}{\left[f\mu_a+(1-f)\mu_b\right]^3}\\
\frac{\partial F}{\partial \epsilon_a} &= \frac{\mu_b+\beta_b r+\epsilon_b}{\left\{[f\mu_a+(1-f)\mu_b]+[f\beta_a+(1-f)\beta_b]r+[f\epsilon_a+(1-f)\epsilon_b]\right\}^2}\\
\frac{\partial^2 F}{\partial \epsilon_a^2} &= \frac{-2f(\mu_b+\beta_b r+\epsilon_b)}{\left\{[f\mu_a+(1-f)\mu_b]+[f\beta_a+(1-f)\beta_b]r+[f\epsilon_a+(1-f)\epsilon_b]\right\}^3} \overset{(0,0,0)}{=\!=} \frac{-2f\mu_b}{\left[f\mu_a+(1-f)\mu_b\right]^3}\\
\frac{\partial F}{\partial \epsilon_b} &= -\frac{\mu_a+\beta_a r+\epsilon_a}{\left\{[f\mu_a+(1-f)\mu_b]+[f\beta_a+(1-f)\beta_b]r+[f\epsilon_a+(1-f)\epsilon_b]\right\}^2}\\
\frac{\partial^2 F}{\partial \epsilon_b^2} &= \frac{2(1-f)(\mu_a+\beta_a r+\epsilon_a)}{\left\{[f\mu_a+(1-f)\mu_b]+[f\beta_a+(1-f)\beta_b]r+[f\epsilon_a+(1-f)\epsilon_b]\right\}^3} \overset{(0,0,0)}{=\!=} \frac{2(1-f)\mu_a}{\left[f\mu_a+(1-f)\mu_b\right]^3}.
\end{aligned}$$

Therefore,

$$\mathbb{E}\left[F(r,\epsilon_a,\epsilon_b)\right] \approx \frac{\mu_a-\mu_b}{f\mu_a+(1-f)\mu_b}+\frac{1}{2}\frac{\partial^2 F_0}{\partial r^2}\mathbb{E}(r^2)-\frac{f\mu_b\mathbb{E}[\epsilon_a^2]}{\left[f\mu_a+(1-f)\mu_b\right]^3}+\frac{(1-f)\mu_a\mathbb{E}[\epsilon_b^2]}{\left[f\mu_a+(1-f)\mu_b\right]^3}.$$

Rearranging terms gives the result. □

Proof of Proposition 6. The condition described in Lemma 2 is a quadratic function of both β_a and β_b. Simple calculations of the vertex suffice to prove the result. □

Proof of Proposition 7. This follows directly from Lemma 2. □

Proof of Proposition 8. This follows from the same derivations in the proof of Proposition 1, with X_a replaced by $\psi(X_a)$ and X_b replaced by $\psi(X_b)$. □

Proof of Proposition 9. In general, the proof follows the same derivations as Lemma 1 and the proofs of Propositions 2–4, though replacing X_a by $\psi(X_a)$ and X_b by $\psi(X_b)$ added substantial analytical complexity.

Let $z \equiv \psi(X_a)/\psi(X_b)$, so that

$$\mathbb{E}[z]=\mathbb{E}\left[\frac{\psi(X_a)}{\psi(X_b)}\right]=\mathbb{E}\left[\frac{\psi(\mu_a+\beta_a r+\epsilon_a)}{\psi(\mu_b+\beta_b r+\epsilon_b)}\right], \tag{A.12}$$

$$\mathbb{E}[1/z]=\mathbb{E}\left[\frac{\psi(X_b)}{\psi(X_a)}\right]=\mathbb{E}\left[\frac{\psi(\mu_b+\beta_b r+\epsilon_b)}{\psi(\mu_a+\beta_a r+\epsilon_a)}\right]. \tag{A.13}$$

We focus on the case where style b dominates the population ($f_\psi^*=0$), which happens when $\mathbb{E}[z]<1$. In other words, we need to identify conditions for which $\mathbb{E}[z]$ tends to decrease. The case where style b dominates the population ($f_\psi^*=1$) is completely symmetric.

First, it is easy to see that $\mathbb{E}[z]$ is an increasing function of μ_a and a decreasing function of μ_b. Similarly, $\mathbb{E}[1/z]$ is a decreasing function of μ_a and an increasing function of μ_b, which proves case (i) of Proposition 9.

To prove case (ii) and (iii), we apply the Taylor approximation of z as a function of r, ϵ_a and ϵ_b to obtain

$$\begin{aligned}
z(r,\epsilon_a,\epsilon_b) &= \frac{\psi(X_a)}{\psi(X_b)}=\frac{\psi(\mu_a+\beta_a r+\epsilon_a)}{\psi(\mu_b+\beta_b r+\epsilon_b)}\\
&= z(0,0,0)+\frac{\partial z_0}{\partial r}r+\frac{\partial z_0}{\partial \epsilon_a}\epsilon_a+\frac{\partial z_0}{\partial \epsilon_b}\epsilon_b\\
&\quad+\frac{1}{2}\left(\frac{\partial^2 z_0}{\partial r^2}r^2+\frac{\partial^2 z_0}{\partial \epsilon_a^2}\epsilon_a^2+\frac{\partial^2 z_0}{\partial \epsilon_b^2}\epsilon_b^2+2\frac{\partial^2 z_0}{\partial r\partial \epsilon_a}r\epsilon_a+2\frac{\partial^2 z_0}{\partial r\partial \epsilon_b}r\epsilon_b+2\frac{\partial^2 z_0}{\partial \epsilon_a\partial \epsilon_b}\epsilon_a\epsilon_b\right)+o(r^2,\epsilon_a^2,\epsilon_b^2).
\end{aligned}$$

After taking the expected value of z, the linear terms vanish, because $\mathbb{E}[r]=\mathbb{E}[\epsilon_a]=\mathbb{E}[\epsilon_b]=0$. The second-order cross terms also vanish because r, ϵ_a and ϵ_b are independent. Therefore, $\mathbb{E}[z]$ can be approximated by $z(0,0,0)$ and the second-order terms:

$$\mathbb{E}[z]=\mathbb{E}\left[\frac{\psi(X_a)}{\psi(X_b)}\right]\approx\frac{\psi(\mu_a)}{\psi(\mu_b)}+\frac{1}{2}\left(\frac{\partial^2 z_0}{\partial r^2}\mathrm{Var}(r)+\frac{\partial^2 z_0}{\partial \epsilon_a^2}\mathrm{Var}(\epsilon_a)+\frac{\partial^2 z_0}{\partial \epsilon_b^2}\mathrm{Var}(\epsilon_b)\right).$$

We then calculate second-order derivatives of $z(r,\epsilon_a,\epsilon_b)$. For simplicity, we use $(0,0,0)$ to represent $r=\epsilon_a=\epsilon_b=0$.

$$\begin{aligned}
\frac{\partial z}{\partial r}&=\frac{\beta_a\psi'(X_a)\psi(X_b)-\beta_b\psi'(X_b)\psi(X_a)}{\psi^2(X_b)}\\
\frac{\partial^2 z}{\partial r^2}&=\frac{\left[\beta_a^2\psi''(X_a)\psi(X_b)-\beta_b^2\psi''(X_b)\psi(X_a)\right]\psi(X_b)-2\beta_b\left[\beta_a\psi'(X_a)\psi(X_b)-\beta_b\psi'(X_b)\psi(X_a)\right]\psi'(X_b)}{\psi^3(X_b)}\\
&\overset{(0,0,0)}{=\!=}\frac{\left[\beta_a^2\psi''(\mu_a)\psi(\mu_b)-\beta_b^2\psi''(\mu_b)\psi(\mu_a)\right]\psi(\mu_b)-2\beta_b\left[\beta_a\psi'(\mu_a)\psi(\mu_b)-\beta_b\psi'(\mu_b)\psi(\mu_a)\right]\psi'(\mu_b)}{\psi^3(\mu_b)}\\
\frac{\partial z}{\partial \epsilon_a}&=\frac{\psi'(X_a)}{\psi(X_b)}\\
\frac{\partial^2 z}{\partial \epsilon_a^2}&=\frac{\psi''(X_a)}{\psi(X_b)}\overset{(0,0,0)}{=\!=}\frac{\psi''(\mu_a)}{\psi(\mu_b)}\\
\frac{\partial z}{\partial \epsilon_b}&=-\frac{\psi(X_a)\psi'(X_b)}{\psi^2(X_b)}\\
\frac{\partial^2 z}{\partial \epsilon_b^2}&=\frac{\psi(X_a)\left[2\left(\psi'(X_b)\right)^2-\psi''(X_b)\psi(X_b)\right]}{\psi(X_b)^3}\overset{(0,0,0)}{=\!=}\frac{\psi(\mu_a)\left[2\left(\psi'(\mu_b)\right)^2-\psi''(\mu_b)\psi(\mu_b)\right]}{\psi(\mu_b)^3}.
\end{aligned}$$

We note that the derivations above reduce to our results in Section 4 when ψ is the identity function. For general ψ that satisfies Assumptions 5–6, it is easy to see that $\frac{\partial^2 z}{\partial \epsilon_a^2}\le 0$ and $\frac{\partial^2 z}{\partial \epsilon_b^2}\ge 0$, which proves case (iii)(a) of Proposition 9.

Next, we analyze $\frac{\partial^2 z}{\partial r^2}$ to prove the remaining part of Proposition 9. First,

$$\begin{aligned}
&\frac{\partial^2 z}{\partial r^2}>0\\
\Longrightarrow\ &\left[\beta_a^2\psi''(\mu_a)\psi(\mu_b)-\beta_b^2\psi''(\mu_b)\psi(\mu_a)\right]\psi(\mu_b)-2\beta_b\left[\beta_a\psi'(\mu_a)\psi(\mu_b)-\beta_b\psi'(\mu_b)\psi(\mu_a)\right]\psi'(\mu_b)>0\\
\Longrightarrow\ &\beta_b^2\psi(\mu_a)\left[2\left(\psi'(\mu_b)\right)^2-\psi(\mu_b)\psi''(\mu_b)\right]+\beta_a\psi(\mu_b)\left[\beta_a\psi(\mu_b)\psi''(\mu_a)-2\beta_b\psi'(\mu_a)\psi'(\mu_b)\right]>0\\
\Longrightarrow\ &\frac{\psi(\mu_a)}{\beta_a}\left[2\left(\psi'(\mu_b)\right)^2-\psi(\mu_b)\psi''(\mu_b)\right]>\frac{\psi(\mu_b)}{\beta_b}\left[2\psi'(\mu_a)\psi'(\mu_b)-\frac{\beta_a}{\beta_b}\psi(\mu_b)\psi''(\mu_a)\right]\\
\Longrightarrow\ &\frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b}>\frac{2\left(\psi'(\mu_b)\right)^2-\psi(\mu_b)\psi''(\mu_b)}{2\psi'(\mu_a)\psi'(\mu_b)-\frac{\beta_a}{\beta_b}\psi(\mu_b)\psi''(\mu_a)}.
\end{aligned}$$

When we consider the symmetric case of $\mathbb{E}[1/z]$, style a and style b are exchanged, so the last inequality becomes:

$$\frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b}<\frac{2\psi'(\mu_b)\psi'(\mu_a)-\frac{\beta_b}{\beta_a}\psi(\mu_a)\psi''(\mu_b)}{2\left(\psi'(\mu_a)\right)^2-\psi(\mu_a)\psi''(\mu_a)}\equiv C_2.$$

Note that C_2 reduces to 1 when ψ is the identity function. This proves case (iii)(b–c) of Proposition 9.

Second, we note that $\frac{\partial^2 z}{\partial r^2}$, and therefore $\mathbb{E}[z]$, is a quadratic function of both β_a and β_b. With respect to β_a, the coefficient of the quadratic term is

$$\psi''(\mu_a)\psi^2(\mu_b)\le 0,$$

and the coefficient of the linear term is

$$-2\beta_b\psi'(\mu_a)\psi'(\mu_b)\psi(\mu_b)\le 0.$$

Therefore, $\mathbb{E}[z]$ is a decreasing function of β_a when β_a is non-negative. This proves case (ii)(a) of Proposition 9 (when we consider the symmetric case of $\mathbb{E}[1/z]$).

With respect to β_b, the coefficient of the quadratic term is

$$-\psi''(\mu_b)\psi(\mu_a)\psi(\mu_b)+2\left(\psi'(\mu_b)\right)^2\psi(\mu_a)\ge 0,$$

and the coefficient of the linear term is

$$-2\beta_a\psi'(\mu_a)\psi'(\mu_b)\psi(\mu_b)\le 0.$$

Therefore, $\mathbb{E}[z]$ achieves its minimum at its vertex:

$$\beta_b = \frac{\beta_a \psi'(\mu_a)\psi'(\mu_b)\psi(\mu_b)}{\left[2\left(\psi'(\mu_b)\right)^2 - \psi''(\mu_b)\psi(\mu_b)\right]\psi(\mu_a)}$$
$$\Longrightarrow \frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b} = \frac{\psi'(\mu_a)\psi'(\mu_b)}{2\left(\psi'(\mu_b)\right)^2 - \psi''(\mu_b)\psi(\mu_b)}.$$

When we consider the symmetric case of $\mathbb{E}[1/z]$, style a and style b are exchanged, so the last equality becomes:

$$\frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b} = \frac{2\left(\psi'(\mu_a)\right)^2 - \psi''(\mu_a)\psi(\mu_a)}{\psi'(\mu_b)\psi'(\mu_a)} \equiv C_1.$$

Note that C_1 reduces to 2 when ψ is the identity function. This proves case (ii)(b-c) of Proposition 9, and therefore completes the proof of the entire proposition. □

Proof of Proposition 10. The comparative statics results with respect to μ_a do not depend on ψ'', so case (i) directly follows. For case (ii), we observe that in the proof of Proposition 9,

$$\frac{\partial^2 z}{\partial \epsilon_a^2} = \frac{\psi''(X_a)}{\psi(X_b)} \overset{(0,0,0)}{=\!=} \frac{\psi''(\mu_a)}{\psi(\mu_b)} > 0$$

when $\psi'' > 0$, which completes the proof. □

Proof of Proposition 11. When the evolutionary equilibrium philosophy involves both investment styles, f_ψ^* is given by the first-order condition, (12). For notational convenience, we let:

$$F(r,\epsilon_a,\epsilon_b) \equiv \frac{\psi(X_a)-\psi(X_b)}{f\psi(X_a)+(1-f)\psi(X_b)} = \frac{\psi(\mu_a+\beta_a r+\epsilon_a)-\psi(\mu_b+\beta_b r+\epsilon_b)}{f\psi(\mu_a+\beta_a r+\epsilon_a)+(1-f)\psi(\mu_b+\beta_b r+\epsilon_b)}.$$

The first-order condition reduces to $\mathbb{E}\left[F(r,\epsilon_a,\epsilon_b)\right]=0$. It is easy to verify that $F(r,\epsilon_a,\epsilon_b)$ is a decreasing function of f. Therefore, we need to identify conditions that lead to higher values of $F(r,\epsilon_a,\epsilon_b)$, which then leads to higher values of f_ψ^* holding other factors constant.

We first calculate the partial derivatives of $F(r,\epsilon_a,\epsilon_b)$ with respect to μ_a and μ_b:

$$\begin{aligned}
\frac{\partial F}{\partial \mu_a} &= \frac{\psi'(X_a)\left[f\psi(X_a)+(1-f)\psi(X_b)\right] - f\psi'(X_a)\left(\psi(X_a)-\psi(X_b)\right)}{\left[f\psi(X_a)+(1-f)\psi(X_b)\right]^2}\\
&= \frac{\psi'(X_a)\psi(X_b)}{\left[f\psi(X_a)+(1-f)\psi(X_b)\right]^2} \geq 0,\\
\frac{\partial F}{\partial \mu_b} &= \frac{-\psi'(X_b)\left[f\psi(X_a)+(1-f)\psi(X_b)\right] + f\psi'(X_b)\left(\psi(X_a)-\psi(X_b)\right)}{\left[f\psi(X_a)+(1-f)\psi(X_b)\right]^2}\\
&= -\frac{\psi'(X_b)\psi(X_b)}{\left[f\psi(X_a)+(1-f)\psi(X_b)\right]^2} \leq 0.
\end{aligned}$$

This proves case (i) of Proposition 11.

To derive further comparative statics, we again use a Taylor expansion to approximate the first-order condition:

$$\mathbb{E}\left[F(r,\epsilon_a,\epsilon_b)\right] \approx \frac{\psi(\mu_a)-\psi(\mu_b)}{f\psi(\mu_a)+(1-f)\psi(\mu_b)} + \frac{1}{2}\left(\frac{\partial^2 F_0}{\partial r^2}\mathrm{Var}(r) + \frac{\partial^2 F_0}{\partial \epsilon_a^2}\mathrm{Var}(\epsilon_a) + \frac{\partial^2 F_0}{\partial \epsilon_b^2}\mathrm{Var}(\epsilon_b)\right).$$

It suffices to calculate the second-order derivatives of $F(r,\epsilon_a,\epsilon_b)$:

$$\begin{aligned}
\frac{\partial F}{\partial r} &= \frac{\beta_a\psi'(X_a)\psi(X_b) - \beta_b\psi(X_a)\psi'(X_b)}{\left[f\psi(X_a)+(1-f)\psi(X_b)\right]^2}\\
\frac{\partial^2 F}{\partial r^2} &\overset{(0,0,0)}{=\!=} \frac{N_1}{\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right]^2}\\
\frac{\partial F}{\partial \epsilon_a} &= \frac{\psi'(X_a)\psi(X_b)}{\left[f\psi(X_a)+(1-f)\psi(X_b)\right]^2}\\
\frac{\partial^2 F}{\partial \epsilon_a^2} &\overset{(0,0,0)}{=\!=} \frac{\psi''(\mu_a)\psi(\mu_b)\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right] - 2f\left(\psi'(\mu_a)\right)^2\psi(\mu_b)}{\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right]^2}\\
\frac{\partial F}{\partial \epsilon_b} &= -\frac{\psi(X_a)\psi'(X_b)}{\left[f\psi(X_a)+(1-f)\psi(X_b)\right]^2}
\end{aligned}$$

$$\frac{\partial^2 F}{\partial \epsilon_b^2} \overset{(0,0,0)}{=\!=} \frac{-\psi(\mu_a)\psi''(\mu_b)\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right]+2(1-f)\psi(\mu_a)\left(\psi'(\mu_b)\right)^2}{\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right]^2}.$$

Here

$$N_1=\left[\beta_a^2\psi''(\mu_a)\psi(\mu_b)-\beta_b^2\psi(\mu_a)\psi''(\mu_b)\right]\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right]-2\left[\beta_a\psi'(\mu_a)\psi(\mu_b)-\beta_b\psi(\mu_a)\psi'(\mu_b)\right]\left[f\beta_a\psi'(\mu_a)+(1-f)\beta_b\psi'(\mu_b)\right].$$

We note that the derivations above reduce to our results in Section 4 when ψ is the identity function. For general ψ that satisfies Assumptions 5–6, it is easy to see that $\frac{\partial^2 F}{\partial \epsilon_a^2}\leq 0$ and $\frac{\partial^2 F}{\partial \epsilon_b^2}\geq 0$, which proves case (iii)(a-b) of Proposition 11.

Next, we analyze $\frac{\partial^2 F}{\partial r^2}$ to prove the remaining part of Proposition 11. First,

$$\begin{aligned}
&\frac{\partial^2 F}{\partial r^2}>0 \Longrightarrow N_1>0\\
\Longrightarrow\ & \left[\beta_a^2\psi''(\mu_a)\psi(\mu_b)-\beta_b^2\psi(\mu_a)\psi''(\mu_b)\right]\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right]\\
&>2\left[\beta_a\psi'(\mu_a)\psi(\mu_b)-\beta_b\psi(\mu_a)\psi'(\mu_b)\right]\left[f\beta_a\psi'(\mu_a)+(1-f)\beta_b\psi'(\mu_b)\right]\\
\Longrightarrow\ & \left[f\beta_a^2\psi''(\mu_a)\psi(\mu_b)-f\beta_b^2\psi(\mu_a)\psi''(\mu_b)\right]\psi(\mu_a)+\left[(1-f)\beta_a^2\psi''(\mu_a)\psi(\mu_b)-(1-f)\beta_b^2\psi(\mu_a)\psi''(\mu_b)\right]\psi(\mu_b)\\
&>\left[2f\beta_a^2\left(\psi'(\mu_a)\right)^2+2(1-f)\beta_a\beta_b\psi'(\mu_a)\psi'(\mu_b)\right]\psi(\mu_b)-\left[2f\beta_a\beta_b\psi'(\mu_a)\psi'(\mu_b)+2(1-f)\beta_b^2\left(\psi'(\mu_b)\right)^2\right]\psi(\mu_a)\\
\Longrightarrow\ & \psi(\mu_a)\left[f\beta_a^2\psi''(\mu_a)\psi(\mu_b)-f\beta_b^2\psi(\mu_a)\psi''(\mu_b)+2f\beta_a\beta_b\psi'(\mu_a)\psi'(\mu_b)+2(1-f)\beta_b^2\left(\psi'(\mu_b)\right)^2\right]\\
&>\psi(\mu_b)\left[2f\beta_a^2\left(\psi'(\mu_a)\right)^2+2(1-f)\beta_a\beta_b\psi'(\mu_a)\psi'(\mu_b)-(1-f)\beta_a^2\psi''(\mu_a)\psi(\mu_b)+(1-f)\beta_b^2\psi(\mu_a)\psi''(\mu_b)\right]\\
\Longrightarrow\ & \frac{\psi(\mu_a)}{\beta_a}\left[f\frac{\beta_a}{\beta_b}\psi''(\mu_a)\psi(\mu_b)-f\frac{\beta_b}{\beta_a}\psi(\mu_a)\psi''(\mu_b)+2f\psi'(\mu_a)\psi'(\mu_b)+2(1-f)\frac{\beta_b}{\beta_a}\left(\psi'(\mu_b)\right)^2\right]\\
&>\frac{\psi(\mu_b)}{\beta_b}\left[2f\left(\psi'(\mu_a)\right)^2+2(1-f)\frac{\beta_b}{\beta_a}\psi'(\mu_a)\psi'(\mu_b)-(1-f)\psi''(\mu_a)\psi(\mu_b)+(1-f)\frac{\beta_b^2}{\beta_a^2}\psi(\mu_a)\psi''(\mu_b)\right]\\
\Longrightarrow\ & \frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b}>C_4,
\end{aligned}$$

where

$$C_4\equiv\frac{2f\left(\psi'(\mu_a)\right)^2+2(1-f)\frac{\beta_b}{\beta_a}\psi'(\mu_a)\psi'(\mu_b)-(1-f)\psi''(\mu_a)\psi(\mu_b)+(1-f)\frac{\beta_b^2}{\beta_a^2}\psi(\mu_a)\psi''(\mu_b)}{f\frac{\beta_a}{\beta_b}\psi''(\mu_a)\psi(\mu_b)-f\frac{\beta_b}{\beta_a}\psi(\mu_a)\psi''(\mu_b)+2f\psi'(\mu_a)\psi'(\mu_b)+2(1-f)\frac{\beta_b}{\beta_a}\left(\psi'(\mu_b)\right)^2}.$$

Note that C_4 reduces to 1 when ψ is the identity function. This proves case (iii)(c–d) of Proposition 11.

Second, we note that $\frac{\partial^2 F}{\partial r^2}$, and therefore the first order condition F, is a quadratic function of both β_a and β_b.

With respect to β_a, the coefficient of the quadratic term is

$$\psi''(\mu_a)\psi(\mu_b)\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right]-2f\left(\psi'(\mu_a)\right)^2\psi(\mu_b)\leq 0,$$

and the coefficient of the linear term is

$$2\beta_b\psi'(\mu_a)\psi'(\mu_b)\left(f\psi(\mu_a)-(1-f)\psi(\mu_b)\right).$$

Therefore, the first order condition achieves its maximum at its vertex:

$$\begin{aligned}
&\beta_a=-\frac{\beta_b\psi'(\mu_a)\psi'(\mu_b)\left(f\psi(\mu_a)-(1-f)\psi(\mu_b)\right)}{\psi''(\mu_a)\psi(\mu_b)\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right]-2f\left(\psi'(\mu_a)\right)^2\psi(\mu_b)}\\
\Longrightarrow\ & \frac{\beta_a}{\psi(\mu_a)}=\frac{\beta_b\psi'(\mu_a)\psi'(\mu_b)\left(f-(1-f)\frac{\psi(\mu_b)}{\psi(\mu_a)}\right)}{\psi(\mu_b)\left\{2f\left(\psi'(\mu_a)\right)^2-\psi''(\mu_a)\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right]\right\}}\\
\Longrightarrow\ & \frac{\psi(\mu_a)/\beta_a}{\psi(\mu_b)/\beta_b}=\frac{2f\left(\psi'(\mu_a)\right)^2-\psi''(\mu_a)\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right]}{\psi'(\mu_a)\psi'(\mu_b)\left(f-(1-f)\frac{\psi(\mu_b)}{\psi(\mu_a)}\right)}\equiv C_3.
\end{aligned}$$

The derivation with respect to β_b follows similarly, which would yield C_3' by simply exchanging terms that correspond to style a and style b. We note that the last equation reduces to the results in Proposition 6 when ψ is the identity function. This proves case (ii) of Proposition 11, and therefore completes the proof of the entire proposition. □

Proof of Proposition 12. The comparative statics results with respect to μ_a and μ_b do not depend on ψ'', so cases (i)–(ii) directly follow. For cases (iii)–(iv), we observe that in the proof of Proposition 11,

$$\frac{\partial^2 F}{\partial \epsilon_a^2} \overset{(0,0,0)}{=\!=} \frac{\psi''(\mu_a)\psi(\mu_b)\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right]-2f\left(\psi'(\mu_a)\right)^2\psi(\mu_b)}{\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right]^2}>0$$
$$\Longleftrightarrow \psi''(\mu_a) > \frac{2f\left(\psi'(\mu_a)\right)^2}{f\psi(\mu_a)+(1-f)\psi(\mu_b)}.$$

Similarly,

$$\frac{\partial^2 F}{\partial \epsilon_b^2} \overset{(0,0,0)}{=\!=} \frac{-\psi(\mu_a)\psi''(\mu_b)\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right]+2(1-f)\psi(\mu_a)\left(\psi'(\mu_b)\right)^2}{\left[f\psi(\mu_a)+(1-f)\psi(\mu_b)\right]^2}<0$$
$$\Longleftrightarrow \psi''(\mu_b) > \frac{2(1-f)\left(\psi'(\mu_b)\right)^2}{f\psi(\mu_a)+(1-f)\psi(\mu_b)}.$$

Both of these conditions can be satisfied by properly choosing a constant lower bound for ψ'', such as:

$$C_5 \equiv \frac{2\max\{\psi'(\mu_a),\psi'(\mu_b)\}^2}{\min\{\psi(\mu_a),\psi(\mu_b)\}}, \tag{A.14}$$

which completes the proof. □

Proof of Proposition 13. The market clearing conditions of Equation (19) yield:

$$\begin{aligned} \frac{W_S\lambda_t}{W_F} &= \left(P_{a,t}/\tilde{P}_{a,t}\right)^k \Longrightarrow P_{a,t}=\tilde{P}_{a,t}\left(\frac{W_S\lambda_t}{W_F}\right)^{\frac{1}{k}},\\ \frac{W_S(1-\lambda_t)}{W_F} &= \left(P_{b,t}/\tilde{P}_{b,t}\right)^k \Longrightarrow P_{b,t}=\tilde{P}_{b,t}\left(\frac{W_S(1-\lambda_t)}{W_F}\right)^{\frac{1}{k}}. \end{aligned} \tag{A.15}$$

In addition, the return processes in Equations (13)–(14) yield:

$$\begin{aligned} R_{at} &= \frac{P_{a,t}}{P_{a,t-1}} = \frac{\tilde{P}_{a,t}\left(\frac{W_S\lambda_t}{W_F}\right)^{\frac{1}{k}}}{\tilde{P}_{a,t-1}\left(\frac{W_S\lambda_{t-1}}{W_F}\right)^{\frac{1}{k}}} = X_{a,t}\left(\frac{\lambda_t}{\lambda_{t-1}}\right)^{\frac{1}{k}},\\ R_{bt} &= \frac{P_{b,t}}{P_{b,t-1}} = \frac{\tilde{P}_{b,t}\left(\frac{W_S(1-\lambda_t)}{W_F}\right)^{\frac{1}{k}}}{\tilde{P}_{b,t-1}\left(\frac{W_S(1-\lambda_{t-1})}{W_F}\right)^{\frac{1}{k}}} = X_{b,t}\left(\frac{1-\lambda_t}{1-\lambda_{t-1}}\right)^{\frac{1}{k}}, \end{aligned} \tag{A.16}$$

where we use the convention that $0/0=1$. This convention is innocuous because in the boundary cases when the aggregate demand stays as a constant, $\lambda_t=0$ or 1, there is no change in demand from the speculators and therefore fundamentalists will drive the return to equal the fundamental value. An alternative way to avoid $0/0$ is to add a small constant demand to both styles in the specification in Equation (17), so that in the boundary cases the aggregate demand in either asset does not vanish. This will not change the equilibrium prices and returns in any essential way. □

Proof of Proposition 14. The equilibrium philosophy f^e is given by Proposition 1 with X_a and X_b replaced by R_a and R_b, and λ_t replaced by f^e, because the aggregate demand must equal the dominant philosophy in equilibrium. The terms $\left(\frac{\lambda_t}{\lambda_{t-1}}\right)^{\frac{1}{k}}$ and $\left(\frac{1-\lambda_t}{1-\lambda_{t-1}}\right)^{\frac{1}{k}}$ in Equation (21) then vanish, and the results follow. □

Proof of Proposition A.1. This follows directly from Lemmas 1–2 and Assumption 7. □

Proof of Proposition A.2. Note that Equation (A.4) is a concave function with respect to $p_1,\cdots,p_m$, so a local maximum is the global maximum. Now suppose $\mathbf{p}^*=(p_1^*,\cdots,p_m^*)$ is a local maximum, then a necessary and sufficient condition is that if we move $\mathbf{p}^*$ toward a direction of any $\mathbf{p}=(p_1,\cdots,p_m)$, the growth rate decreases. Formally, let

$$\mathbf{p}^\delta=(1-\delta)\mathbf{p}^*+\delta\mathbf{p},$$

where $\mathbf{p}$ is arbitrary and $0\le\delta\le1$, and

$$\mu(\mathbf{p}^\delta)=\mathbb{E}\left[\log\left(\left((1-\delta)p_1^*+\delta p_1\right)X_1+\cdots+\left((1-\delta)p_m^*+\delta p_m\right)X_m\right)\right].$$

Then, $\mathbf{p}^*=(p_1^*,\cdots,p_m^*)$ maximizes Equation (A.4) if and only if:

$$\left.\frac{\partial\mu(\mathbf{p}^\delta)}{\partial\delta}\right|_{\delta=0}\le 0,\ \text{for any } \mathbf{p}=(p_1,\cdots,p_m),$$

which further leads to:

$$\mathbb{E}\left[\frac{(p_1-p_1^*)X_1+\cdots+(p_m-p_m^*)X_m}{p_1^*X_1+\cdots+p_m^*X_m}\right]\le 0,\ \text{for any } \mathbf{p}=(p_1,\cdots,p_m)$$
$$\Longrightarrow \mathbb{E}\left[\frac{p_1X_1+\cdots+p_mX_m}{p_1^*X_1+\cdots+p_m^*X_m}\right]\le 1,\ \text{for any } \mathbf{p}=(p_1,\cdots,p_m)$$

which completes the proof. □

Proof of Proposition A.3. The first m conditions in Equation (A.6) follow directly from Proposition A.2. As for the last case, note that $p_1=1-p_2-\cdots-p_m$ and we can write $\mu(\cdot)$ as a function of $(p_2,\cdots,p_m)$. Therefore $\mathbf{p}^*$ is given by the following equations:

$$\begin{cases} \frac{\partial\mu(p_2,\cdots,p_m)}{\partial p_2}\Big|_{p_{l+1}=\cdots=p_m=0}=0 \\ \frac{\partial\mu(p_2,\cdots,p_m)}{\partial p_3}\Big|_{p_{l+1}=\cdots=p_m=0}=0 \\ \cdots \\ \frac{\partial\mu(p_2,\cdots,p_m)}{\partial p_l}\Big|_{p_{l+1}=\cdots=p_m=0}=0. \end{cases} \tag{A.17}$$

Also, the following partial derivatives must be negative:

$$\begin{cases} \frac{\partial\mu(p_2,\cdots,p_m)}{\partial p_{l+1}}\Big|_{\mathbf{p}^*}<0 \\ \cdots \\ \frac{\partial\mu(p_2,\cdots,p_m)}{\partial p_m}\Big|_{\mathbf{p}^*}<0. \end{cases} \tag{A.18}$$

Equation (A.17) yields

$$\mathbb{E}\left[\frac{X_1}{p_1X_1+\cdots+p_lX_l}\right]=\cdots=\mathbb{E}\left[\frac{X_l}{p_1X_1+\cdots+p_lX_l}\right].$$

Suppose that the value above equals C, then

$$1=\mathbb{E}\left[\frac{p_1X_1+\cdots+p_lX_l}{p_1X_1+\cdots+p_lX_l}\right]=(p_1+\cdots+p_l)C=C.$$

Equation (A.18) yields

$$\mathbb{E}\left[\frac{X_j}{p_1X_1+\cdots+p_lX_l}\right]<\mathbb{E}\left[\frac{X_1}{p_1X_1+\cdots+p_lX_l}\right]=1$$

for $j=l+1,l+2,\cdots,m$. which completes the proof. □

Proof of Proposition A.4. We observe in Proposition A.3 that the conditions for style 1-investors to dominate resemble closely the condition for style a-investors to dominate in Proposition 1. Therefore, the proof of comparative statics follows directly from generalizations of Propositions 2–4. In particular, Cases (i)–(ii) follow from the fact that $\mathbb{E}\left[\frac{X_k}{X_1}\right]$ is a decreasing function of μ_1 and increasing function of μ_k for $k=2,3,\ldots,m$. Cases (iii)–(viii) follow from generalizing results in Equation (A.11) into $m-1$ ratios of style returns. □

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