

# PDE 小型学术研讨会

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时间	报告题目	报告人
8:20—8:30 开幕		
主持：章志飞		
8:30-9:00	三维不可压缩欧拉方程小截面涡环解的轨道稳定性	秦国林
9:00-9:30	Eventual continuity approach to verifying unique ergodicity of SPDEs	刘子愉
9:30-10:00	Global uniqueness of incompressible inhomogeneous Navier-Stokes equations with bounded density	郝田田
10:00-10:30	茶歇	
主持：桂贵龙		
10:30-11:00	Some results on global manifolds for dissipative PDEs	赵嘉成
11:00-11:45	A relative entropy approach to the phase transition problems	刘豫宁
午餐(12:00-14:00)		
主持：章志飞		
14:00-18:00	自由讨论	

# 报告摘要

## 三维不可压缩欧拉方程小截面涡环解的轨道稳定性

秦国林（中国科学院数学院）

涡环解是三维不可压缩欧拉方程一类特殊的稳态解。自 1970 年 Fraenkel 等人工作后，关于涡环解的存在性已有大量的研究。但是关于涡环解唯一性和稳定性的结果很少。报告人将介绍关于小截面涡环解的唯一性和轨道稳定性的研究。我们的证明基于变分方法和 Lyapunov-Schmidt 约化方法。

## Eventual continuity approach to verifying unique ergodicity of SPDEs

刘子愉（北京大学）

We formulate a new criterion of the asymptotic stability for some non-equicontinuous Markov semigroups, the so-called eventually continuous semigroups. In particular, we provide a non-equicontinuous Markov semigroup example with essential randomness, which is asymptotically stable. We further apply the eventual continuity approach to the study of the ergodicity of stochastic partial differential equations with multiplicative noise. We apply the generalized coupling method to verify the eventual continuity and combine it with the uniform irreducibility to verify the unique ergodicity.

## Global uniqueness of incompressible inhomogeneous Navier-Stokes equations with bounded density

郝田田（中国科学院数学院）

We consider the uniqueness of 3-D incompressible inhomogeneous Navier-Stokes equations in a critical regularity framework, that is  $u_0 \in \dot{B}^{-1+\frac{3}{p}}_{p,1}$  for  $2 < p < 3$ , in the case where the initial density is close to a positive constant in  $L^\infty$  but has no regularity. This result extends the previous results in [Danchin, Wang 2022], here we need additional  $u_0 \in L^n(\mathbb{R}^3)$  for  $n > 3$  instead of  $u_0 \in L^2(\mathbb{R}^3)$ . We also

give a uniqueness statement in the case of  $u_0 \in \{\dot{B}^{-1+\frac{3}{p}}\}_{p,1} \cap L^2 + \{\dot{B}^{-1+\frac{3}{p}}\}_{p,1} \cap L^n$ .

## Some results on global manifolds for dissipative PDEs

赵嘉成 (上海师范大学)

In this talk, I give a brief description of the existence results on finite-dimensional global manifolds for several types of dissipative PDEs, including phase-field systems, FitzHugh-Nagumo systems and non-autonomous hyperdissipative Navier-Stokes equations. These are joint works with Profs. Alain Miranville, Rong-Nian Wang and Jianhong Wu. Finally, some ideas for further research are involved.

## A relative entropy approach to the phase transition problems

刘豫宁 (上海纽约大学)

In this talk we shall first review the relative entropy method by Fischer--Laux--Simon. One feature of the entropy there is that it controls both discrepancy of the Allen-Cahn energy and the calibration of the level-set of the phase field function. It is also compatible with the gradient flow structure so that a Gronwall inequality can be derived without Modica's maximal principle. In the study of vector-valued phase transition problems, it provides estimates of vectorial nature that lead to effective geometric motions when combined with weak convergence methods.